2021年度物理工学実験技法(A)

磁気抵抗と電子構造

物性研究所　長田俊人

§1.　Boltzmann Transport Phenomena　角度依存磁気抵抗振動

1. Boltzmann equation and relaxation time approx.
2. Angular Dependent Magnetoresistance Oscillations (Q2D)
3. Angular Dependent Magnetoresistance Oscillations (Q1D)
4. high frequency / high electric field effects

§2.　Quantum Oscillations　シュブニコフ･ドハース効果

0. Landau levels
1. Lifshitz-Kosevich formula
2. analysis of Shubnikov-de Haas effect
3. Fourier analysis: FFT & MEM
4. magnetic breakdown effect
5. quantum interference effect
6. Berry phase

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1. 角度依存磁気抵抗効果とFermi面

§ 1. Boltzmann Transport Phenomena

1. Boltzmann equation and relaxation time approx.
2. Angular Dependent Magnetoresistance Oscillations (Q2D)
3. Angular Dependent Magnetoresistance Oscillations (Q1D)
4. high frequency / high electric field effects

Boltzmann Equation

- semiclassical electron kinetics in $\mathbf{k}\otimes\mathbf{R}$ phase space
  - semiclassical equation of motion under effective mass approximation

$$\begin{cases} \mathbf{v}_{\mathbf{k}} \equiv \dot{\mathbf{R}} = \dfrac{1}{\hbar}\dfrac{\partial E_{\mathbf{k}}}{\partial \mathbf{k}} \\ \hbar\dot{\mathbf{k}} = \mathbf{F} = -e\mathbf{E} - e\mathbf{v}_{\mathbf{k}} \times \mathbf{B} \end{cases} \qquad \hbar\mathbf{K} \equiv \hbar\mathbf{k} + (-e)\mathbf{A}$$

- Boltzmann equation

$$\frac{\partial f}{\partial t} + \dot{\mathbf{k}} \cdot \frac{\partial f}{\partial \mathbf{k}} + \dot{\mathbf{R}} \cdot \frac{\partial f}{\partial \mathbf{R}} = \left(\frac{\partial f}{\partial t}\right)_{\text{scatt.}}$$

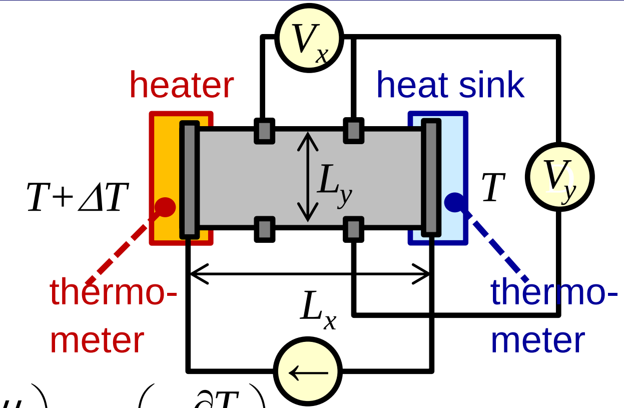
  - distribution function: $f(\mathbf{k},\mathbf{R},t) = f^0(E_{\mathbf{k}}) + \Delta f(\mathbf{k},\mathbf{R},t)$
  - equilibrium distribution: $f^0(E_{\mathbf{k}}) = \dfrac{1}{e^{(E_{\mathbf{k}}-\mu(\mathbf{R}))/k_B T(\mathbf{R})}+1}$
  - linearized Boltzmann equation

$$\frac{\partial \Delta f}{\partial t} + \frac{(-e)}{\hbar}(\mathbf{v}_{\mathbf{k}} \times \mathbf{B}) \cdot \frac{\partial \Delta f}{\partial \mathbf{k}} + \mathbf{v}_{\mathbf{k}} \cdot \frac{\partial \Delta f}{\partial \mathbf{R}} - \left(\frac{\partial \Delta f}{\partial t}\right)_{scatt.} = -\left(\frac{\partial f^0}{\partial E_{\mathbf{k}}}\right)\mathbf{v}_{\mathbf{k}} \cdot \left\{\left((-e)\mathbf{E} - \frac{\partial \mu}{\partial \mathbf{R}}\right) - \frac{E_{\mathbf{k}}-\mu}{T}\frac{\partial T}{\partial \mathbf{R}}\right\}$$

  - relaxation time approximation (elastic scattering) $\left(\dfrac{\partial \Delta f}{\partial t}\right)_{scatt.} = -\dfrac{\Delta f}{\tau_{\mathbf{k}}} = -\dfrac{f-f^0}{\tau_{\mathbf{k}}}$
  - Chambers' kinetic solution (stationary state)

$$\boxed{\Delta f = \int_{-\infty}^{t}\left(-\frac{\partial f^0}{\partial E_{\mathbf{k}}}\right)\mathbf{v}_{\mathbf{k}} \cdot \left((-e)\mathbf{E} - \frac{\partial \mu}{\partial \mathbf{R}} - \frac{E_{\mathbf{k}}-\mu}{T}\frac{\partial T}{\partial \mathbf{R}}\right) e^{\int_t^{t'} \frac{1}{\tau_{\mathbf{k}}} dt''} dt'}$$

Transport Coefficients



- case of constant $E_{\mathbf{k}}(=E)$ and $\tau_{\mathbf{k}}(=\tau)$

$$\Delta f_{\mathbf{k}} = \left(-\frac{\partial f^0}{\partial E_{\mathbf{k}}}\right)\tau\langle \mathbf{v}_{\mathbf{k}}\rangle\cdot\left[(-e)\mathbf{E} - \frac{\partial\mu}{\partial\mathbf{R}} - \frac{E_{\mathbf{k}}-\mu}{T}\frac{\partial T}{\partial \mathbf{R}}\right]$$

$$\langle \mathbf{v}_{\mathbf{k}(0)}\rangle \equiv \int_{-\infty}^{0} \mathbf{v}_{\mathbf{k}(t)} e^{t/\tau} d(t/\tau)$$

- transport coefficients

$$\begin{cases} \mathbf{j} = \dfrac{1}{L^d}\displaystyle\sum_{\mathbf{k},\sigma}(-e)\mathbf{v}_{\mathbf{k}}\Delta f_{\mathbf{k}} = e^2\overleftrightarrow{K}_0\cdot\left(\mathbf{E}+\dfrac{1}{e}\dfrac{\partial\mu}{\partial\mathbf{R}}\right) + e\overleftrightarrow{K}_1\cdot\left(\dfrac{1}{T}\dfrac{\partial T}{\partial\mathbf{R}}\right) \equiv \overleftrightarrow{\sigma}\cdot\left(\mathbf{E}+\dfrac{1}{e}\dfrac{\partial\mu}{\partial\mathbf{R}}\right) + \overleftrightarrow{\alpha}\cdot\left(-\dfrac{\partial T}{\partial\mathbf{R}}\right) \\ \mathbf{j}_Q = \dfrac{1}{L^d}\displaystyle\sum_{\mathbf{k},\sigma}(E_{\mathbf{k}}-\mu)\mathbf{v}_{\mathbf{k}}\Delta f_{\mathbf{k}} = -e\overleftrightarrow{K}_1\cdot\left(\mathbf{E}+\dfrac{1}{e}\dfrac{\partial\mu}{\partial\mathbf{R}}\right) - \overleftrightarrow{K}_2\cdot\left(\dfrac{1}{T}\dfrac{\partial T}{\partial\mathbf{R}}\right) \equiv T\overleftrightarrow{\alpha}'\cdot\left(\mathbf{E}+\dfrac{1}{e}\dfrac{\partial\mu}{\partial\mathbf{R}}\right) + \overleftrightarrow{\kappa}_{\mathbf{E}}\cdot\left(-\dfrac{\partial T}{\partial\mathbf{R}}\right) \end{cases}$$

熱電係数

絶対熱起電力
Seebeck係数
Nernst係数

$$\boxed{\begin{cases} \mathbf{E} = \overleftrightarrow{\rho}\cdot\mathbf{j} + \overleftrightarrow{S}\cdot\dfrac{\partial T}{\partial\mathbf{R}} \\ \mathbf{j}_Q = \overleftrightarrow{\Pi}\cdot\mathbf{j} - \overleftrightarrow{\kappa}\cdot\dfrac{\partial T}{\partial\mathbf{R}} \end{cases}}$$

Peltier係数　　熱伝導度

$$\overleftrightarrow{K}_n \equiv \frac{2}{(2\pi)^d}\int \tau(\mathbf{v}_{\mathbf{k}}\circ\langle\mathbf{v}_{\mathbf{k}}\rangle)(E_{\mathbf{k}}-\mu)^n\left(-\frac{\partial f^0}{\partial E_{\mathbf{k}}}\right)d^d\mathbf{k}$$

$$= \int\left\{\frac{2}{(2\pi)^d}\oint_{E_{\mathbf{k}}=E}\tau(\mathbf{v}_{\mathbf{k}}\circ\langle\mathbf{v}_{\mathbf{k}}\rangle)\frac{d^{d-1}S_{\mathbf{k}}}{\hbar|\mathbf{v}_{\mathbf{k}}|}\right\}(E-\mu)^n\left(-\frac{\partial f^0}{\partial E}\right)dE \equiv \int\frac{\overleftrightarrow{\sigma}_0(E)}{e^2}(E-\mu)^n\left(-\frac{\partial f^0}{\partial E}\right)dE$$

$$\overleftrightarrow{S} = \overleftrightarrow{\sigma}^{-1}\overleftrightarrow{\alpha} = -\frac{\pi^2}{3}\frac{k_B{}^2T}{e}\overleftrightarrow{\sigma}^{-1}\left(\frac{d\overleftrightarrow{\sigma}_0(E)}{dE}\right)_{E=\mu}$$

Mott formula

$$\frac{\kappa}{\sigma T} = L \equiv \frac{\pi^2}{3}\left(\frac{k_B}{e}\right)^2 = 2.44\times10^{-8}[\mathrm{W\Omega/K^2}]$$

Wiedemann-Franz law

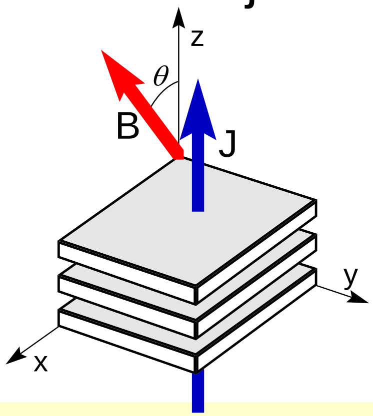
- electric conductivity

Chambers formula

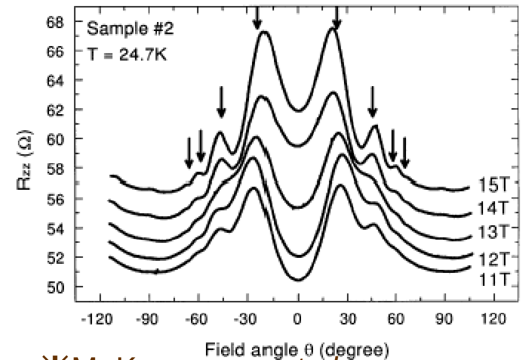
$$\boxed{\sigma_{ij} = \frac{2e^2}{V}\sum_{\mathbf{k}} v_i(\mathbf{k}(0))\left\{\int_{-\infty}^{0} v_j(\mathbf{k}(t))e^{t/\tau}dt\right\}\left(-\frac{\partial f^0}{\partial E_{\mathbf{k}}}\right) = \frac{2e^2}{(2\pi)^d}\int_{\mathrm{BZ}} v_i(\mathbf{k}(0))\left\{\int_{-\infty}^{0} v_j(\mathbf{k}(t))e^{t/\tau}dt\right\}\left(-\frac{\partial f^0}{\partial E_{\mathbf{k}}}\right)d^d\mathbf{k}}$$

Interlayer Magnetotransport in Q2D Layered Conductors

AMRO : Angle-dependent MagnetoResistance Oscillations
("Kartsovnik-Kajita-Yamaji oscillations")

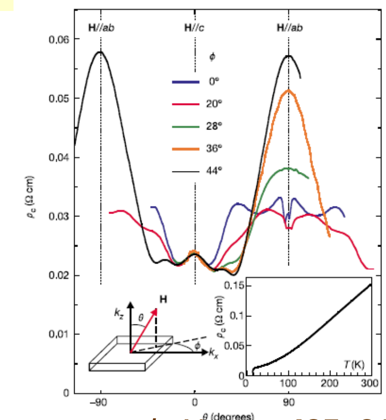


GaAs/AlGaAs superlattice



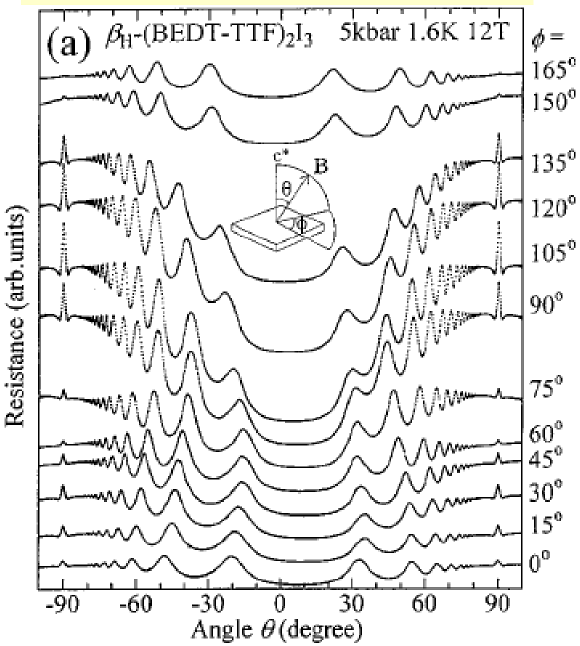
※M. Kawamura *et al.*, PhysicaB**249-251**, 882 (1998).

overdoped $Tl_2Ba_2CuO_{6+\delta}$ (layered high-T_c oxide)



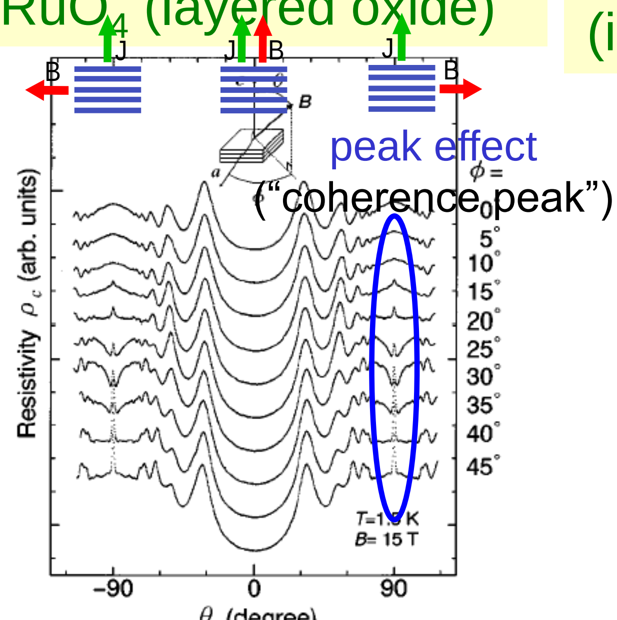
※N. E. Hussey *et al.*, Nature **425**, 814 (2003).

β_H-(BEDT-TTF)$_2$I$_3$ (organic conductor)



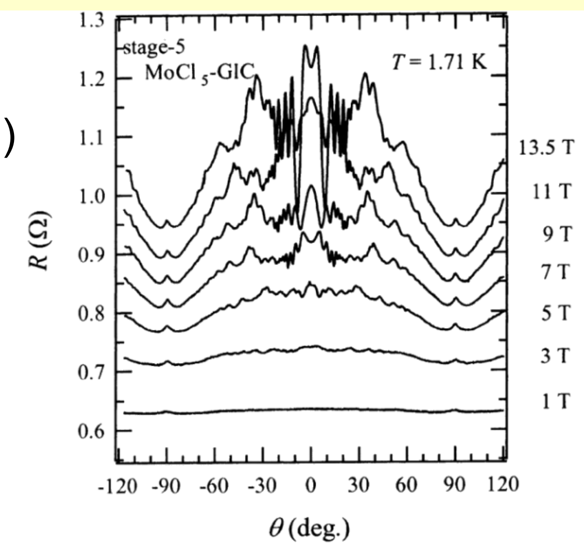
※N. Hanasaki *et al.*, PR**B57**, 1336 (1998).

Sr_2RuO_4 (layered oxide)



※E. Ohmichi *et al.*, PR**B59**, 7263 (1999).

stage-5 $MoCl_5$-GIC (intercalation compound)



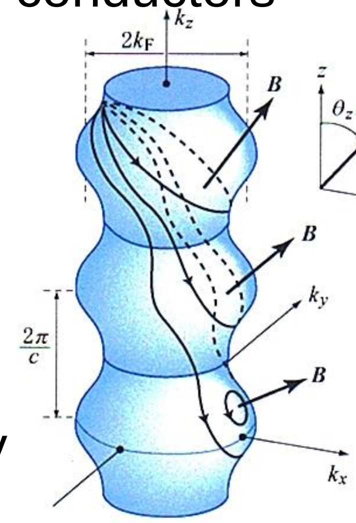
※K. Enomoto *et al.*, Synth. Met. **154**, 289 (2005).

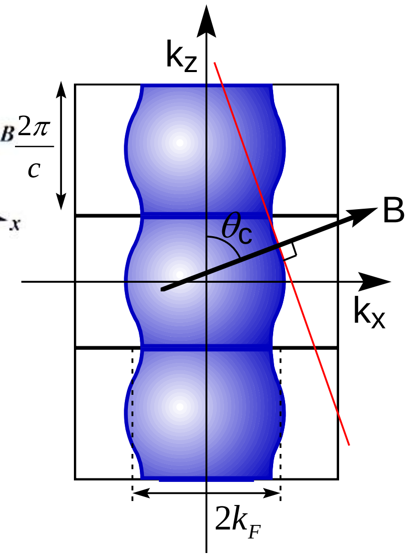
Semiclassical Electron Orbital Motion on Q2D Fermi Surface

- Kartsovnik-Kajita-Yamaji oscillations in Q2D conductors

① band model: $E(\mathbf{k}) = \dfrac{\hbar^2 (k_x{}^2 + k_y{}^2)}{2m} - 2t_c \cos ck_z$

② equation of motion: $\begin{cases} \mathbf{v} = \dot{\mathbf{R}} = \dfrac{1}{\hbar}\dfrac{\partial E(\mathbf{k})}{\partial \mathbf{k}} \\ \hbar\dot{\mathbf{k}} = (-e)\mathbf{v}\times\mathbf{B} \end{cases}$





③ Chambers formula for interlayer conductivity
(Boltzmann eq. + relaxation time approx.)

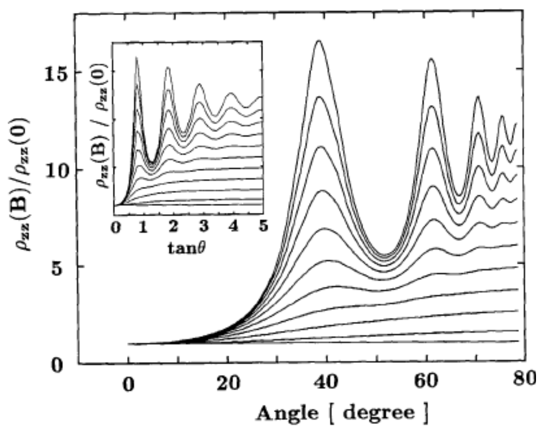
$$\sigma_{zz} = \frac{2e^2}{V}\sum_{\mathbf{k}}\left(-\frac{\partial f^0}{\partial E_{\mathbf{k}}}\right) v_z(\mathbf{k}(0)) \int_{-\infty}^{0} v_z(\mathbf{k}(t)) e^{t/\tau} dt$$

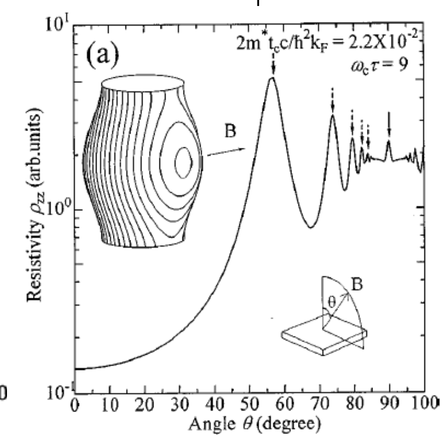
⇩

$$\boxed{\sigma_{zz} = \frac{2t_c{}^2 cme^2}{\pi\hbar^4}\sum_{\nu=-\infty}^{\infty} J_\nu (ck_F \tan\theta)^2 \frac{\tau}{1+(\nu\omega_c\tau)^2}}$$

$$J_\nu(z) \approx \sqrt{\frac{2}{\pi z}} \cos\left(z - \nu\frac{\pi}{2} - \frac{\pi}{4}\right) \qquad (z >> \nu^2)$$

: Bessel function





※R. Yagi, Y. Iye, T.Osada, and S. Kagoshima, J. Phys. Soc. Jpn. **59**, 3069 (1990).

$$\tan\theta_c \equiv \frac{v_F}{\max v_z} = \frac{\left(\frac{\hbar k_F}{m}\right)}{\left(\frac{2t_c c}{\hbar}\right)} = \frac{\hbar^2 k_F}{2mt_c c}$$

- peak effect: large tilt angle $\theta_c \le \theta \le \dfrac{\pi}{2}$

※N. Hanasaki, S. Kagoshima, T. Hasegawa, T. Osada, and N. Miura, Phys. Rev. B**57**, 1336 (1998).

Experimental Determination of Fermi Surface Using AMRO

- AMRO and Fermi surface shape

$$\sigma_{zz} \approx \frac{2t_c^{\,2}cme^2\tau}{\pi\hbar^4} J_0(ck_F \tan\theta)^2$$

$$J_0(z) \approx \sqrt{\frac{2}{\pi z}} \cos\left(z - \frac{\pi}{4}\right) \quad \left(z >> \nu^2\right)$$

resistance peaks of AMRO

⇔ conductivity minima

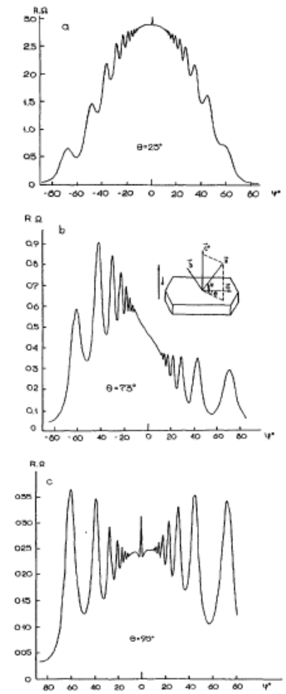
⇔ zero points of $J_0(ck_F\tan\theta)$

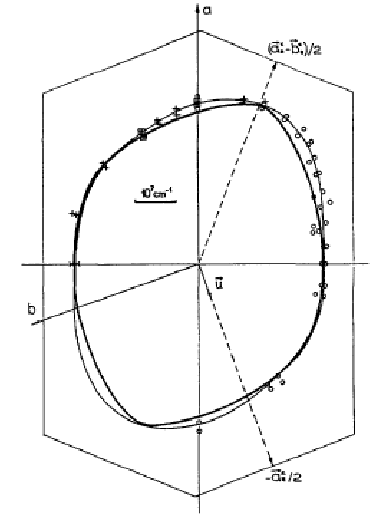
$$\boxed{\tan\theta = \frac{\pi}{ck_F}\left(N - \frac{1}{4}\right)}$$

⇩

shape of the cross section of cylindrical Fermi surface ?

⇩





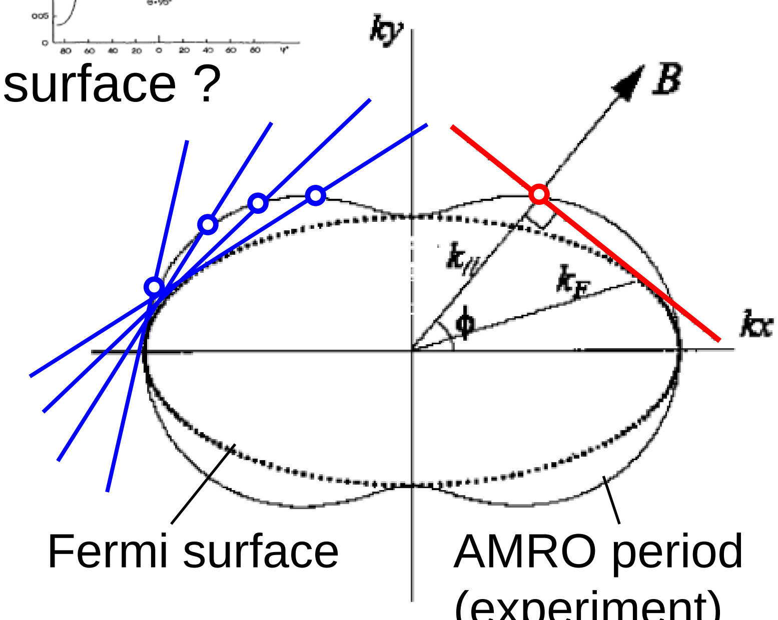
β-(BEDT-TTF)$_2$IBr$_2$

- case of non-cylindrical Fermi surface

$$\boxed{\tan\theta = \frac{\pi}{ck_{//}(\phi)}\left(N - \frac{1}{4}\right)}$$

⇩

envelope of Fermi surface cross section

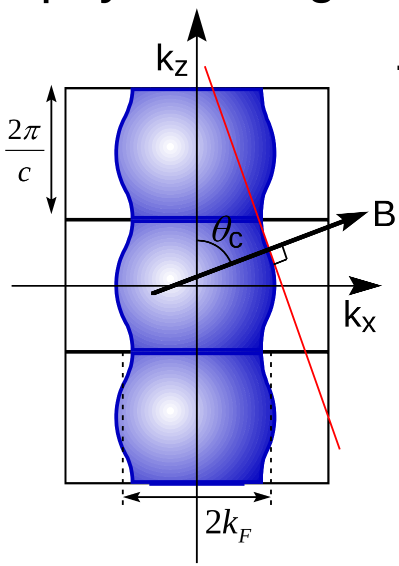


※ M. V. Kartsovnik, V. N. Laughkin, S. I. Psotskii, I. F. Schegolev, and V. M. Yakovenko, J. de Physique I (France) **2**, 89 (1992).

Coherence Peak of Interlayer Magnetoresistance in Q2D Systems

※ N. Hanasaki, S. Kagoshima, T. Hasegawa, T. Osada, and N. Miura, Phys. Rev. B **57**, 1336 (1998).

- physical origin of the peak effect



- large tilt angle: $\theta_c \le \theta \le \frac{\pi}{2}$ $\qquad \tan\theta_c \equiv \frac{v_F}{\max v_z} = \frac{\left(\frac{\hbar k_F}{m}\right)}{\left(\frac{2t_c c}{\hbar}\right)} = \frac{\hbar^2 k_F}{2mt_c c}$

⇓

appearance of fixed points on the Fermi surface $\dot{\mathbf{k}} = -\frac{e}{\hbar}\mathbf{v_k} \times \mathbf{B} = 0$

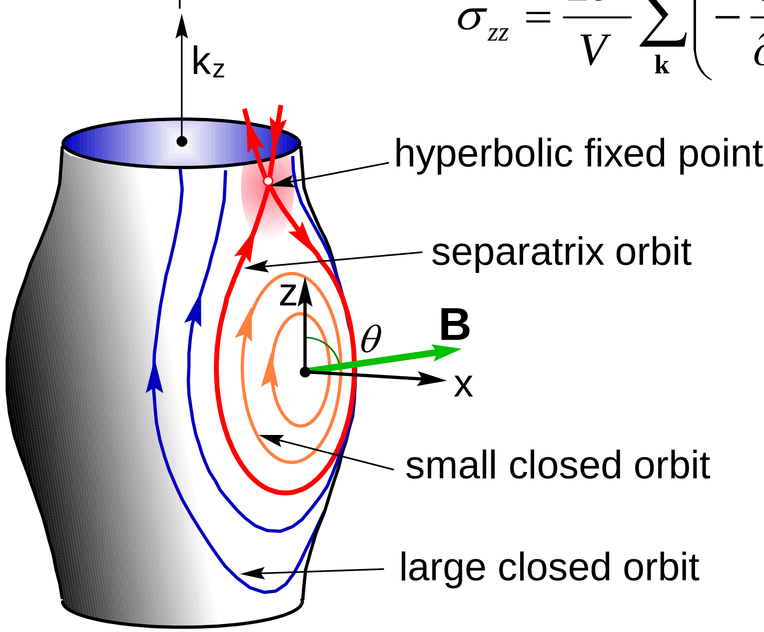
⇓

slow electron motion near the hyperbolic fixed point

⇓

large velocity-velocity correlation ⇒ dominant contribution

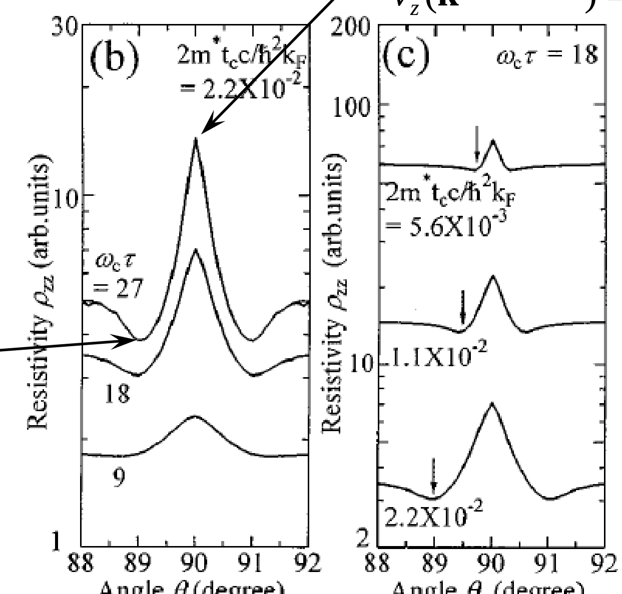
$$\sigma_{zz} = \frac{2e^2}{V}\sum_{\mathbf{k}}\left(-\frac{\partial f^0}{\partial E_{\mathbf{k}}}\right)v_z(\mathbf{k}(0))\int_{-\infty}^{0} v_z(\mathbf{k}(t))e^{t/\tau}dt$$



$\theta = \frac{\pi}{2}$

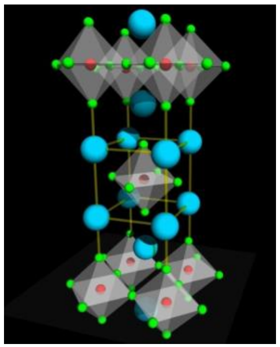
$v_z(\mathbf{k}^{(\text{fixed point})}) = 0$

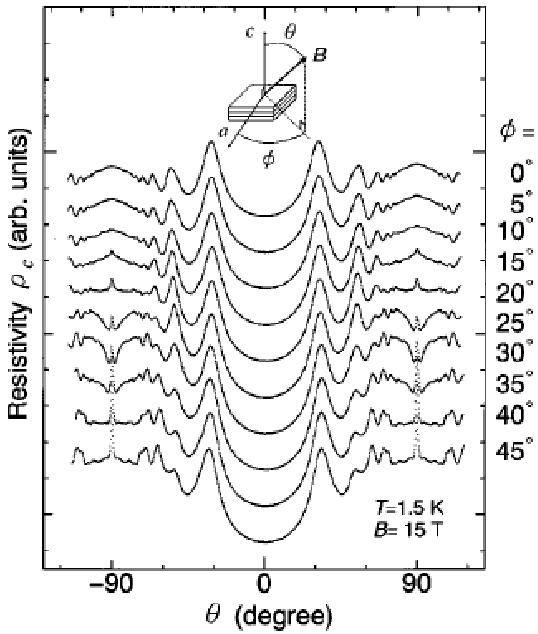
$\theta = \theta_c$

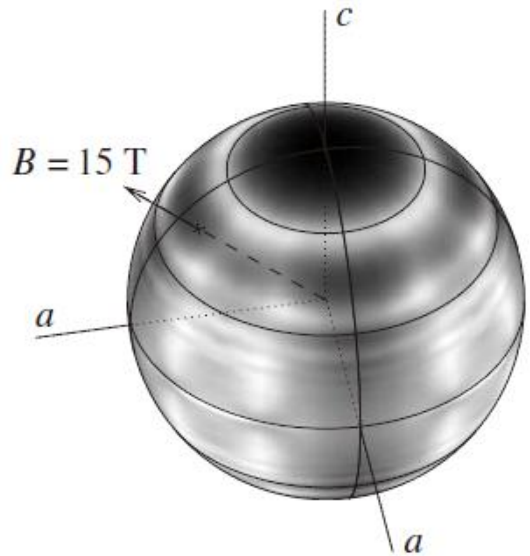


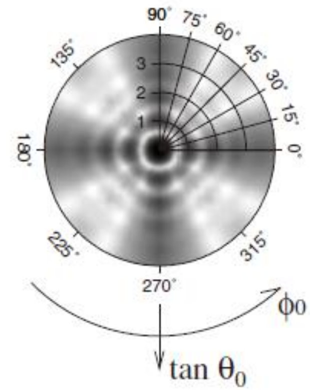
Angle-Depandent Magnetoresistance Oscillation of Sr_2RuO_4

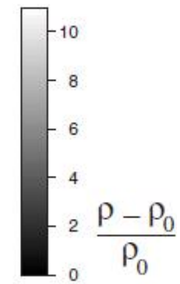
※E. Ohmichi, H. Adachi, Y. Mori, Y. Maeno, T. Ishiguro, and T. Oguchi, Phys. Rev. B **59**, 7263 (1999).
※C. Bergemann, A. P. MacKenzie, S. R. Julian, D. Forsythe, and E. Ohmichi, Adv. Phys. **52**, 639 (2003).

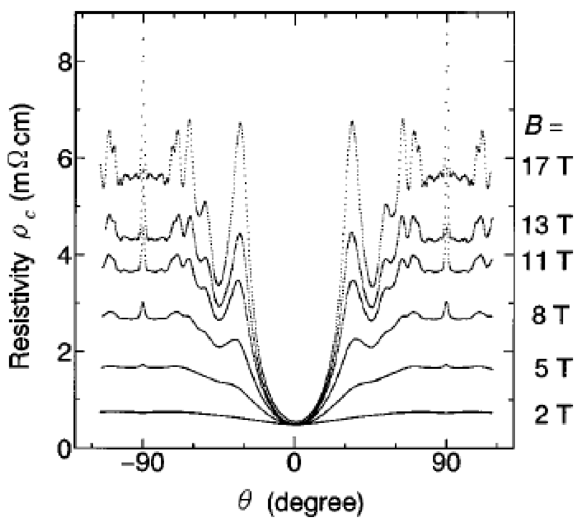


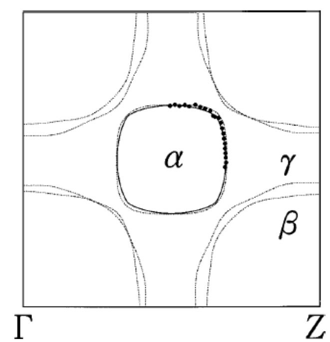


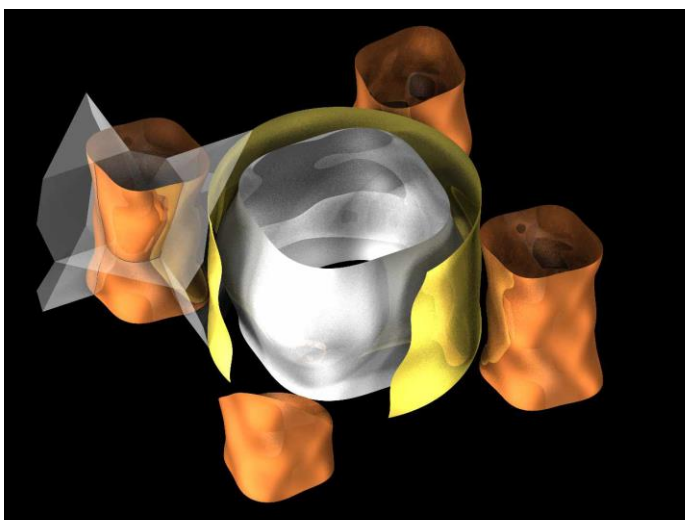






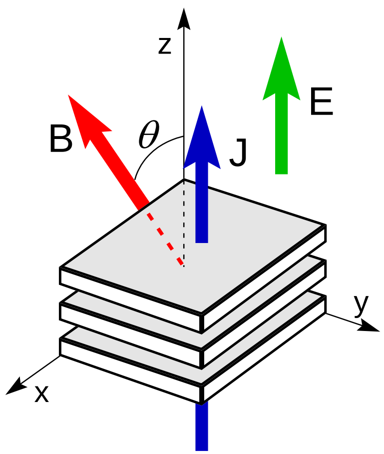






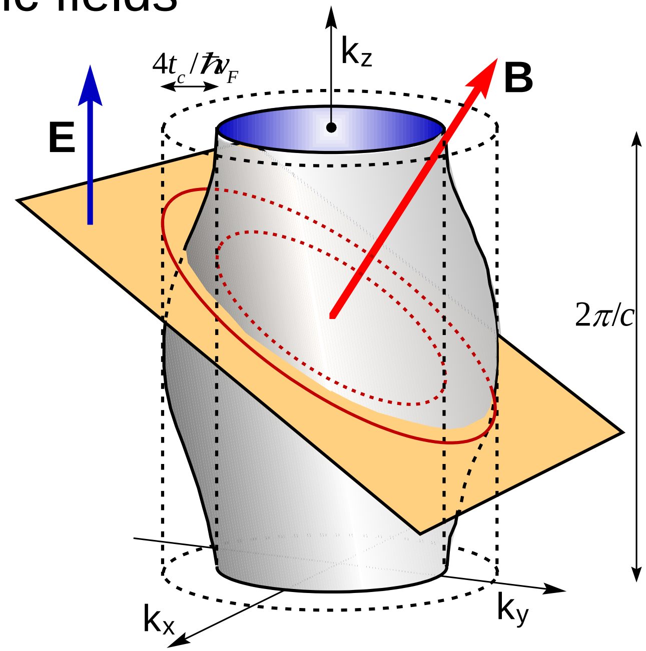
Electron Kinetics and Interlayer Transport under $\mathbf{B}$ and $\mathbf{E}$ Fields

- Electron orbital motion under magnetic and electric fields



$$E(\mathbf{k}) = \frac{\hbar^2\left(k_x{}^2 + k_y{}^2\right)}{2m} - 2t_c \cos ck_z$$

$$\begin{cases} \hbar\dot{\mathbf{k}} = (-e)\mathbf{v}\times\mathbf{B} + (-e)\mathbf{E} \\ \mathbf{v} = \dfrac{1}{\hbar}\dfrac{\partial E(\mathbf{k})}{\partial \mathbf{k}} \end{cases}$$



- electron orbit exists in the cylinder surface with the thickness of $4t_c/hv_F$.
- electron exists on the plane perpendicular to the field moving along the stacking axis with the constant velocity of eE/h.

- Interlayer conduction

$$\mathrm{Re}\left(\frac{j_z}{E_z}\right) \approx \frac{2e^2}{V}\sum_{\mathbf{k}^0}\left(-\frac{df}{dE}\right) v_z\left(\mathbf{k}^0\right)\int_{-\infty}^{0} v_z\left(\mathbf{k}(t)\right) e^{\frac{t}{\tau}}\, dt$$

$$\boxed{\frac{j_z}{E_z} \approx \frac{2t_c{}^2 cme^2}{\pi\hbar^4}\sum_{\nu=-\infty}^{\infty}\frac{\tau J_\nu\left(ck_F \tan\theta\right)^2}{1+\left(\omega_B - \nu\omega_c\right)^2\tau^2}}$$

$\omega_c \equiv \dfrac{eB_z}{m}$: cyclotron frequency

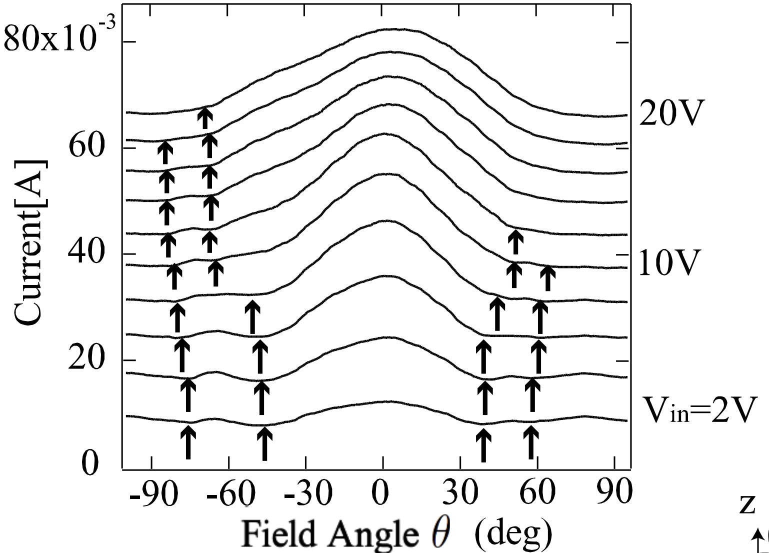
$\omega_B \equiv \dfrac{ceE}{\hbar}$: Bloch frequency

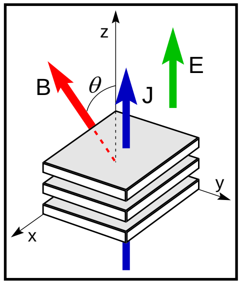
Angle-Dependent Stark Cyclotron Resonance in Q2D Systems

※A. Kumagai, T. Konoike, K. Uchida, and T. Osada, J. Phys. Soc. Jpn. **81**, 023708 (2012).

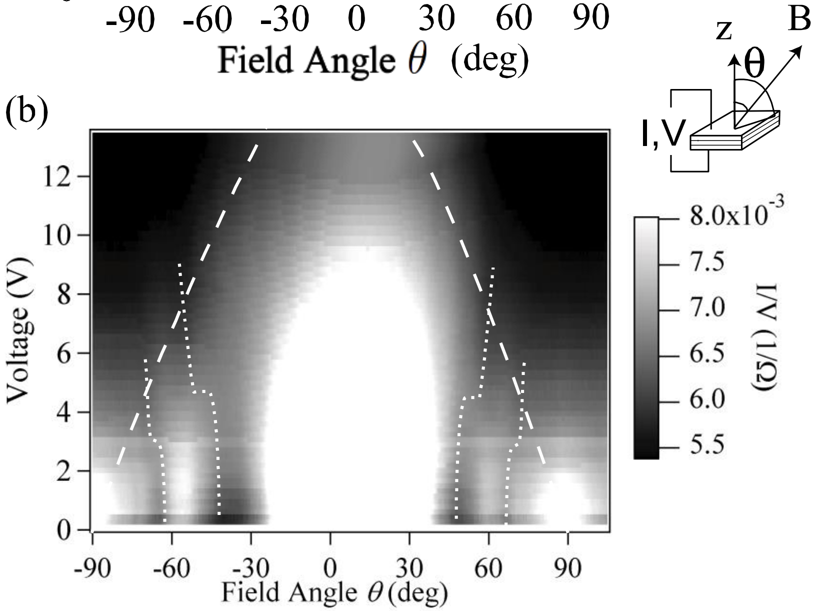
● experiment on Q2D conductor

(a) α-(BEDT-TTF)$_2$(NH$_4$)Hg(SCN)$_4$



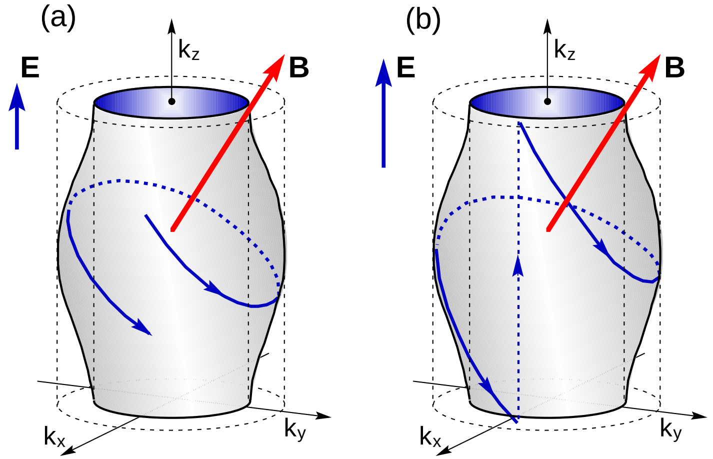


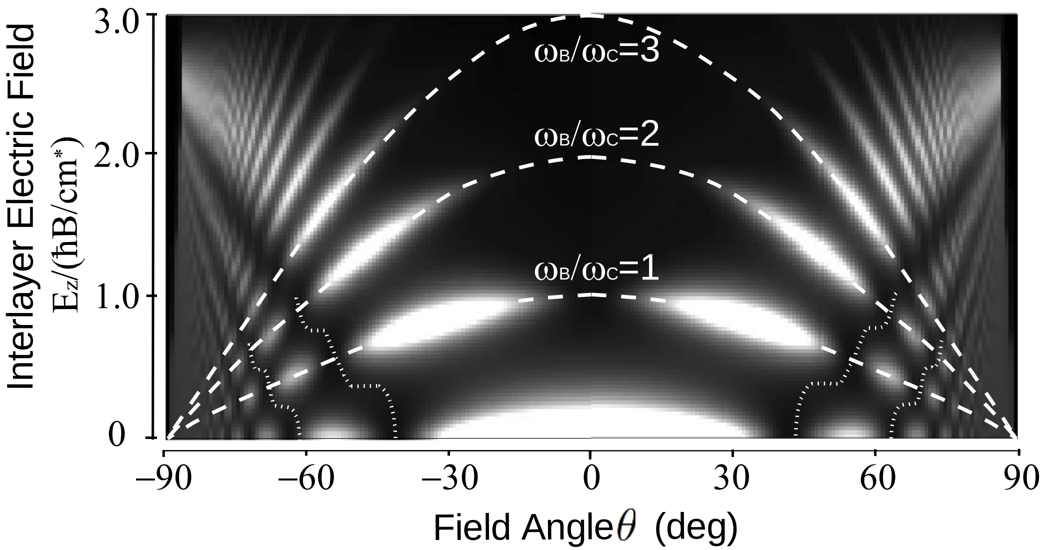
(b)



● theory

$$\frac{j_z}{E_z} \approx \frac{2t_c{}^2 cme^2}{\pi\hbar^4} \sum_{\nu=-\infty}^{\infty} \frac{\tau J_\nu\left(ck_F \tan\theta\right)^2}{1+\left(\omega_B - \nu\omega_c\right)^2 \tau^2}$$

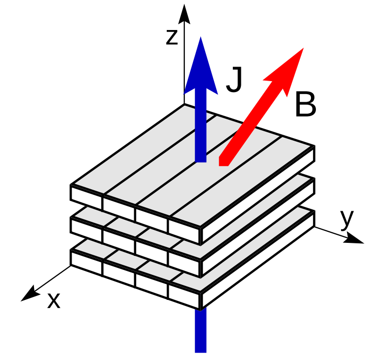




Angle-Dependent Magnetotransport in Q1D Conductors

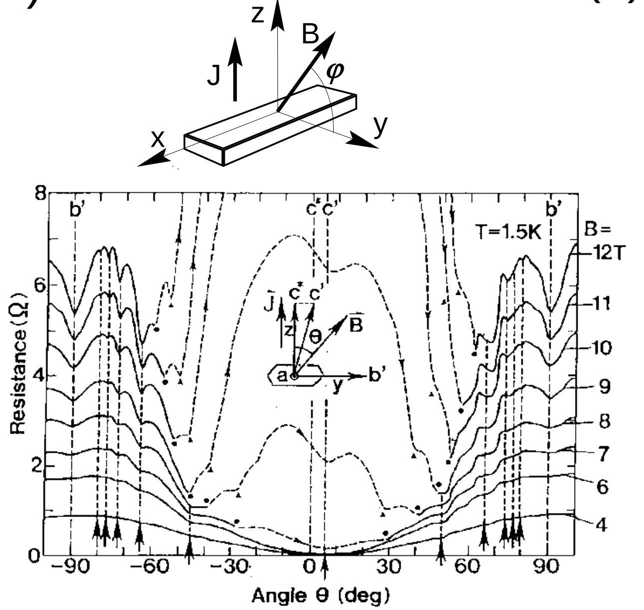
- magnetoresistance angular effects in Q1D conductors
  - interlayer resistance in layered quasi-one-dimensional conductors
  - dependence on the orientation of magnetic fields

(1) Lebed resonances
(2) Danner-Chaikin oscillations
(3) third angular effect



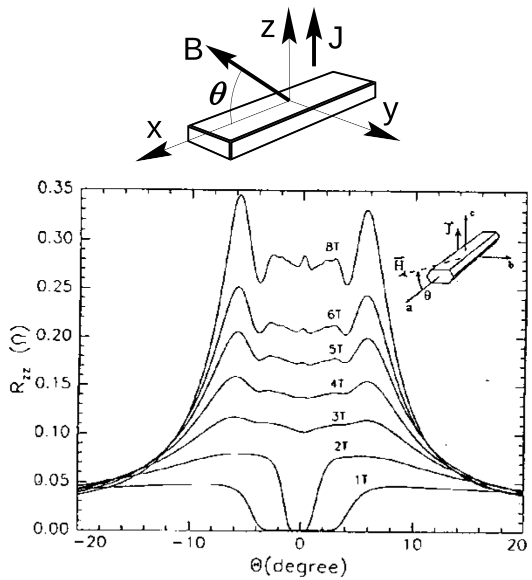
- Q1D organic conductor $(TMTSF)_2ClO_4$

(1) Lebed resonance



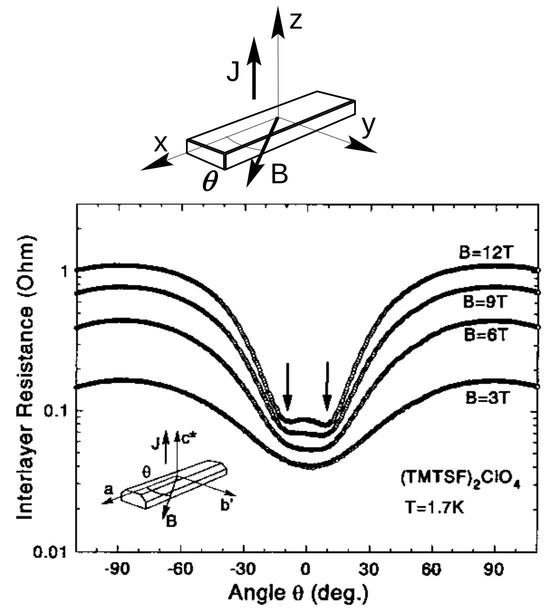
※T. Osada, A. Kawasumi, S. Kagoshima, N. Miura, and G. Saito, Phys. Rev. Lett. **77**, 5261 (1996).

(2) Danner-Chaikin oscillation



※G.M.Danner, W. Kang, and P. M. Chaikin, Phys. Rev. Lett. **72**, 3714 (1994).

(3) third angular effect

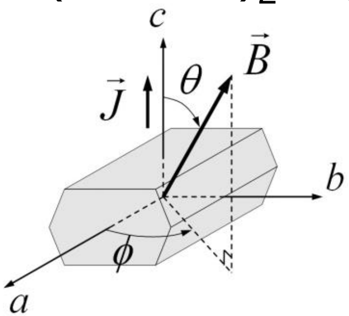


※T. Osada, S. Kagoshima, and N. Miura, Phys. Rev. Lett. **77**, 5261 (1996).

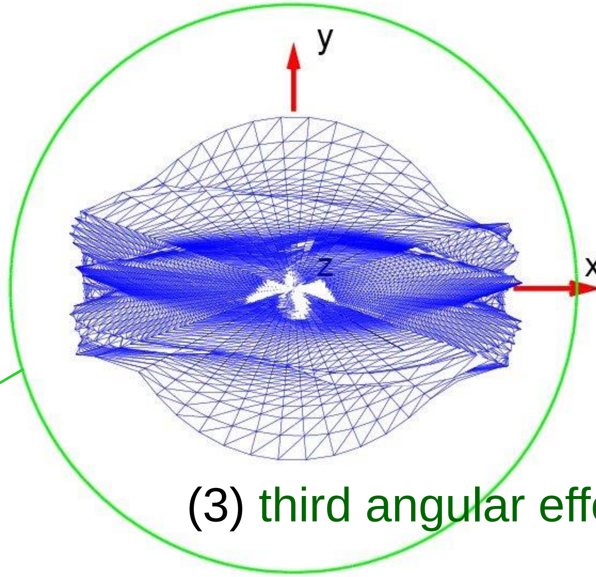
Stereographic Interlayer Magnetoresistance in Q1D Conductor

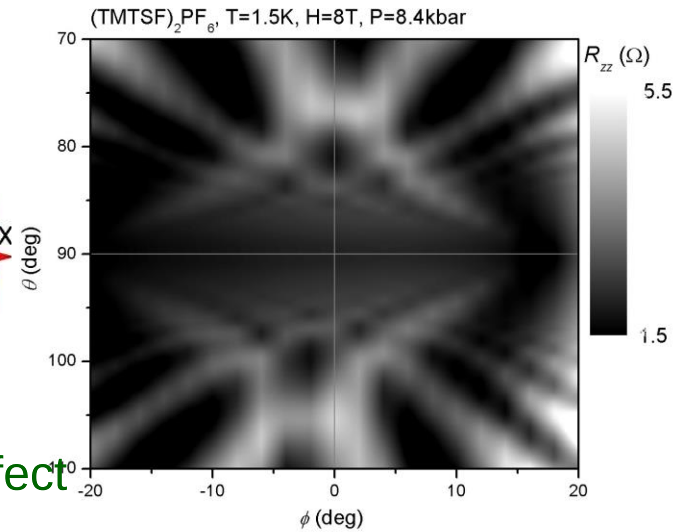
※ W. Kang, T. Osada, Y. J. Jo, and H. Kang, Phys. Rev. Lett. **99**, 017002 (2007).

$(TMTSF)_2PF_6$



T=1.5K
B=8.0T
P=8.4kbar

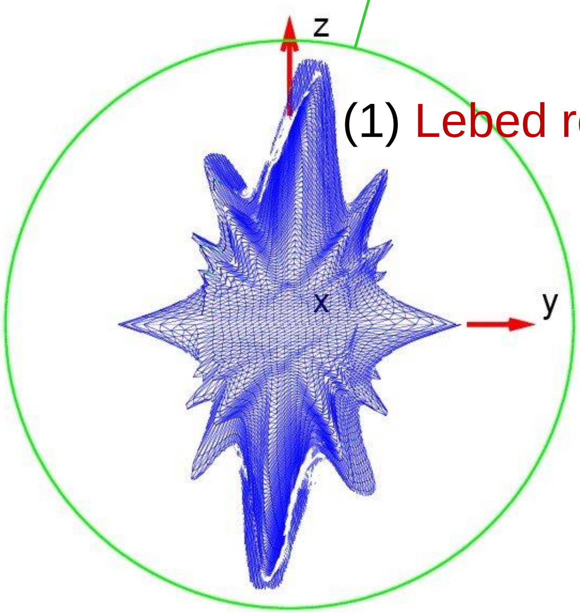




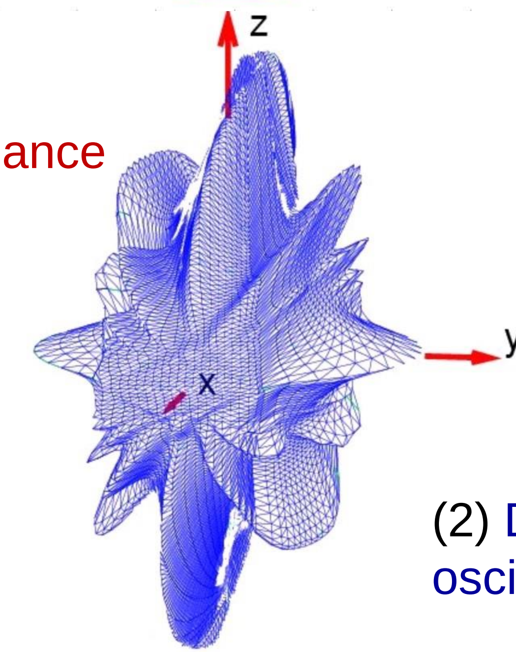
interlayer conductivity
(logarithmic scale)

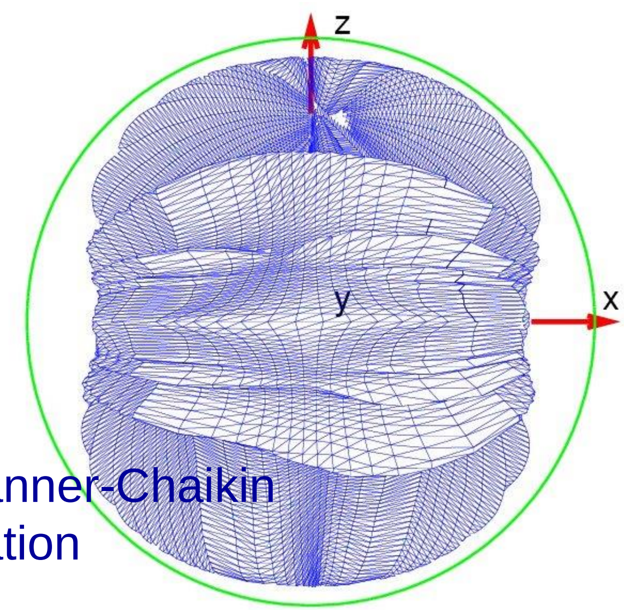
zero field conductivity

(3) third angular effect



(1) Lebed resonance





(2) Danner-Chaikin
oscillation

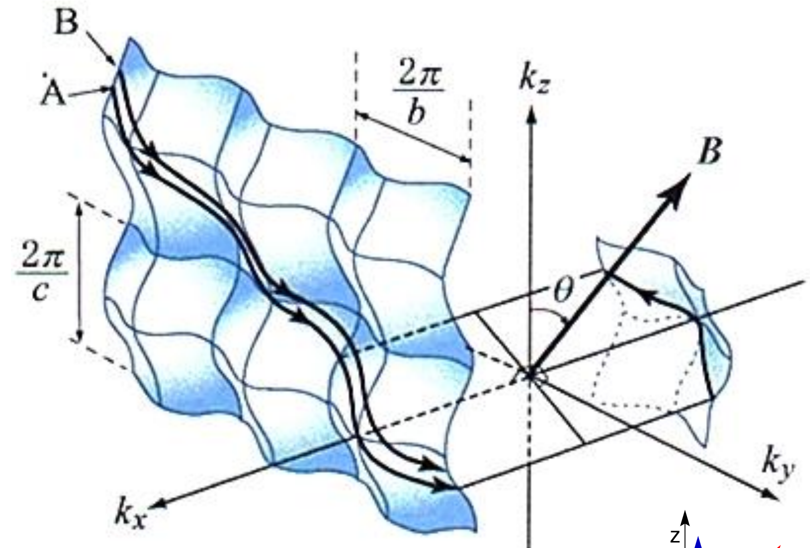
Semiclassical Electron Orbital Motion on Q1D Fermi Surface

① band model:

$$E(\mathbf{k}) = -2t_a \cos ak_x - 2t_b \cos bk_y - 2t_c \cos ck_z - E_F$$

$$(E_F \sim t_a >> t_b >> t_c) \quad \Downarrow$$

$$E(\mathbf{k}) = \hbar v_F (|k_x| - k_F) - 2t_b \cos bk_y - 2t_c \cos ck_z$$



② equation of motion: $\begin{cases} \mathbf{v} = \dot{\mathbf{R}} = \dfrac{1}{\hbar}\dfrac{\partial E(\mathbf{k})}{\partial \mathbf{k}} \\ \hbar\dot{\mathbf{k}} = (-e)\mathbf{v} \times \mathbf{B} \end{cases}$

③ Chambers formula

(←Boltzmann eq. + relaxation time approx.)

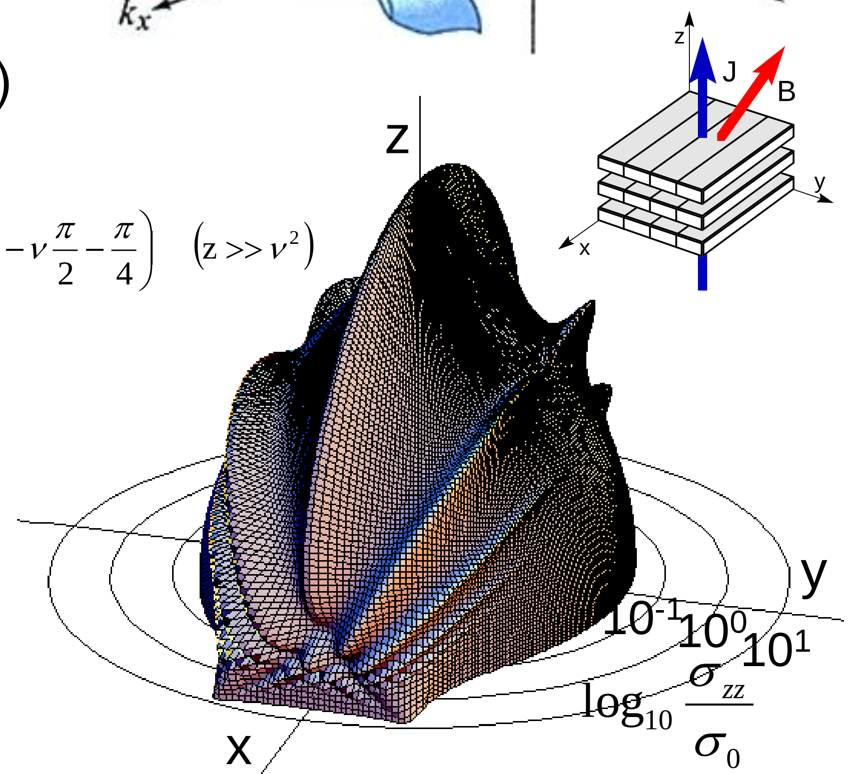
$$\sigma_{zz} = \frac{2e^2}{V}\sum_{\mathbf{k}}\left(-\frac{\partial f^0}{\partial E_{\mathbf{k}}}\right) v_z(\mathbf{k}(0)) \int_{-\infty}^{0} v_z(\mathbf{k}(t)) e^{t/\tau} dt$$

$$\Downarrow$$

$$J_\nu(z) \approx \sqrt{\frac{2}{\pi z}} \cos\left(z - \nu\frac{\pi}{2} - \frac{\pi}{4}\right) \quad (z >> \nu^2)$$

$$\boxed{\sigma_{zz} = \frac{4}{2\pi bc\hbar v_F}\left(\frac{et_c c}{\hbar}\right)^2 \sum_{\pm,\nu} J_\nu(\gamma)^2 \frac{\tau}{1 + \{(\nu \pm \alpha)\Omega\tau\}^2}}$$

$$\alpha \equiv \frac{c}{b}\frac{B_y}{B_z}, \quad \gamma \equiv \frac{2t_b c}{\hbar v_F}\frac{B_x}{B_z}, \quad \Omega \equiv \frac{beB_z}{\hbar} v_F, \quad \alpha\Omega = \frac{ceB_y}{\hbar} v_F$$



(1) Lebed resonance

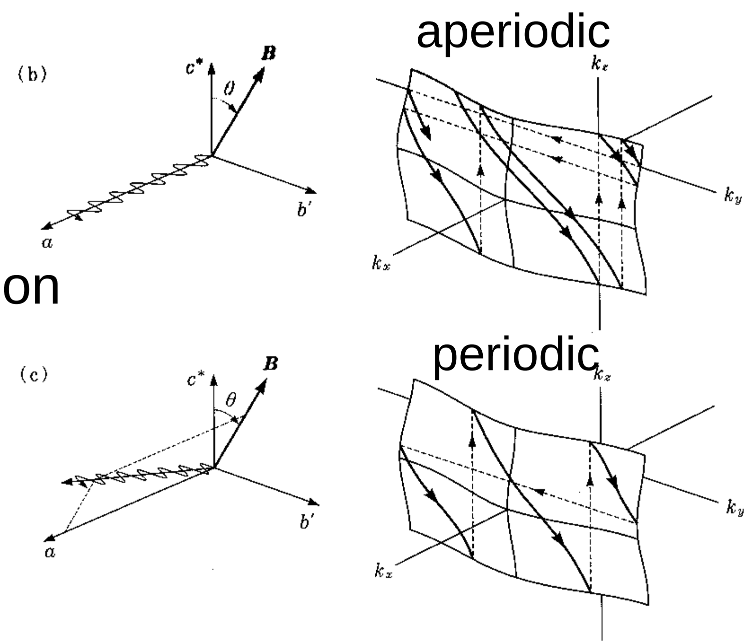
(2) Danner-Chaikin oscillations

(3) third angular effect

(4) no peak effect (← approx. of $|B_z/B_x| >> 2t_c c/hv_F$)

Physical Meanings of Magnetoresistance Angular Effects

$$\sigma_{zz} = \frac{4}{2\pi bc\hbar v_F}\left(\frac{et_c c}{\hbar}\right)^2 \sum_{\pm,\nu} J_\nu(\gamma)^2 \frac{\tau}{1+\{(\nu\pm\alpha)\Omega\tau\}^2}$$



(1) Lebed resonance:

commensurability (periodicity) of orbital motion

$$p\pm\alpha = p\pm\frac{c}{b}\frac{B_y}{B_z}=0 \;\Rightarrow\; \boxed{\tan\theta_{z\to y}=\frac{B_y}{B_z}=p\frac{b}{c}}$$

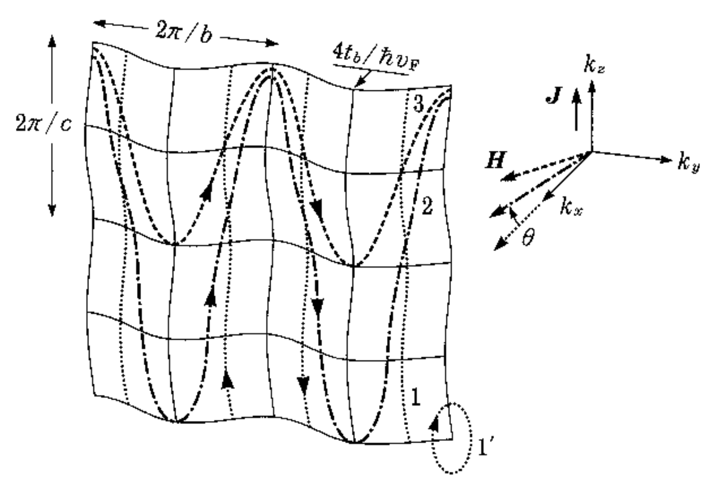
Lebed's magic angles

(2) Danner-Chaikin oscillation: amplitude modulation of Lebed resonance

$$J_p(\gamma)^2 = J_p\left(\frac{2t_b c}{\hbar v_F}\frac{B_x}{B_z}\right)^2 = J_p\left(c\frac{2t_b}{\hbar v_F}\tan\theta\cos\phi\right)^2 = 0$$

- case of x-z plane rotation (ϕ=0)

$$J_0\left(c\frac{2t_b}{\hbar v_F}\tan\theta_{z\to x}\right)^2=0 \;\Longrightarrow\; \tan\theta_{z\to x}=\frac{\pi\hbar v_F}{2t_b c}\left(N-\frac{1}{4}\right)$$



Physical Meanings of Magnetoresistance Angular Effects 2

(3) third angular effect:

accumulation of maximum amplitude of Lebed resonances.

- p-th Lebed resonance at $\dfrac{B_y}{B_z} = p\dfrac{b}{c}$

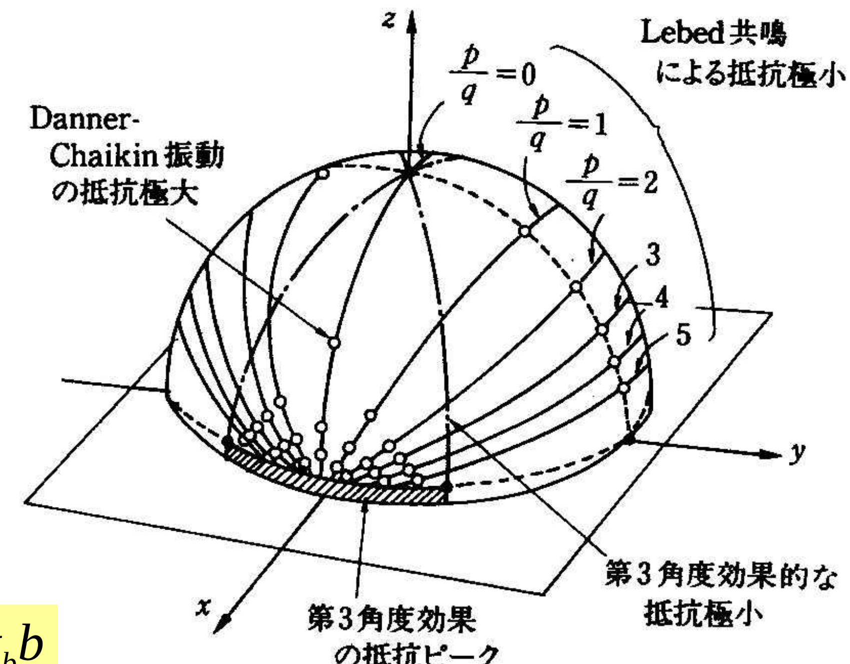
  modulated by the oscillating factor $J_p(\gamma)^2$.

- $J_p(\gamma)^2$ has the maximum peak around

$$\Downarrow \qquad \gamma = \frac{2t_b c}{\hbar v_F}\frac{B_x}{B_z} \approx p$$

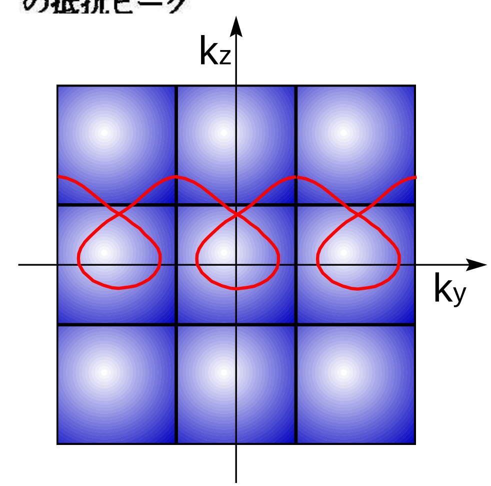
- Each Lebed resonance has the maximum amplitude when $\dfrac{B_y}{B_x} = \dfrac{2t_b b}{\hbar v_F}$.

- particularly on x-y plane, $\tan\theta_{x\to y} = \dfrac{B_y}{B_x} = \dfrac{2t_b b}{\hbar v_F}$



(4) peak effect: appearance of fixed points

$$\left|\tan\theta_{x\to z}\right| \le \frac{2t_c c}{\hbar v_F}$$



Periodic Orbit Resonance (POR)

- a.c. interlayer conductivity

※A. E. Kovale, S. Hill, and J. S. Qualls, Phys. Rev. B **66**, 134513 (2002).

$$\sigma_{zz}(\omega) = \frac{4}{2\pi b c \hbar v_F}\left(\frac{e t_c c}{\hbar}\right)^2 \sum_{\pm,\nu} J_\nu(\gamma)^2 \frac{\tau}{1+\{\omega-(\nu\pm\alpha)\Omega\}^2\tau^2}$$

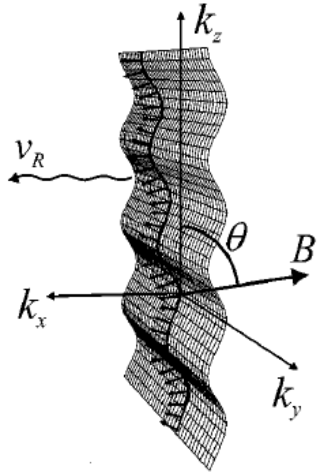
$$\alpha \equiv \frac{c}{b}\frac{B_y}{B_z}, \quad \gamma \equiv \frac{2t_b c}{\hbar v_F}\frac{B_x}{B_z}, \quad \Omega \equiv \frac{beB_z}{\hbar}v_F$$

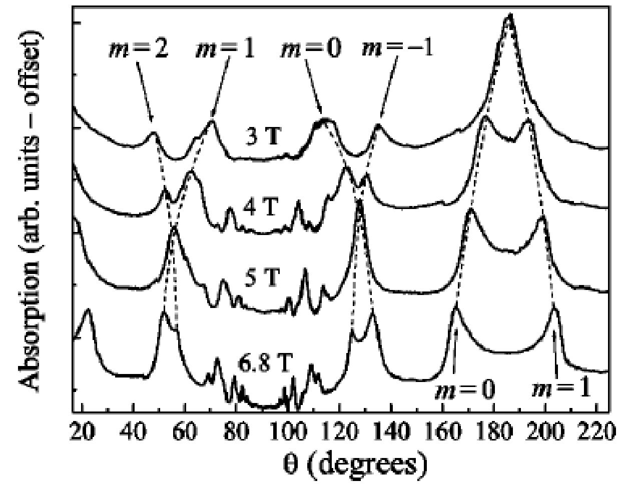
- resonant condition

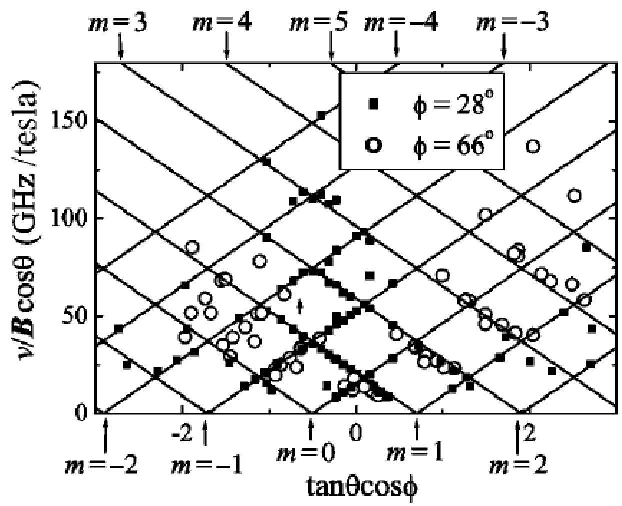
$$\omega = (p\pm\alpha)\Omega = p\frac{beB_z}{\hbar}v_F \pm \frac{ceB_y}{\hbar}v_F$$

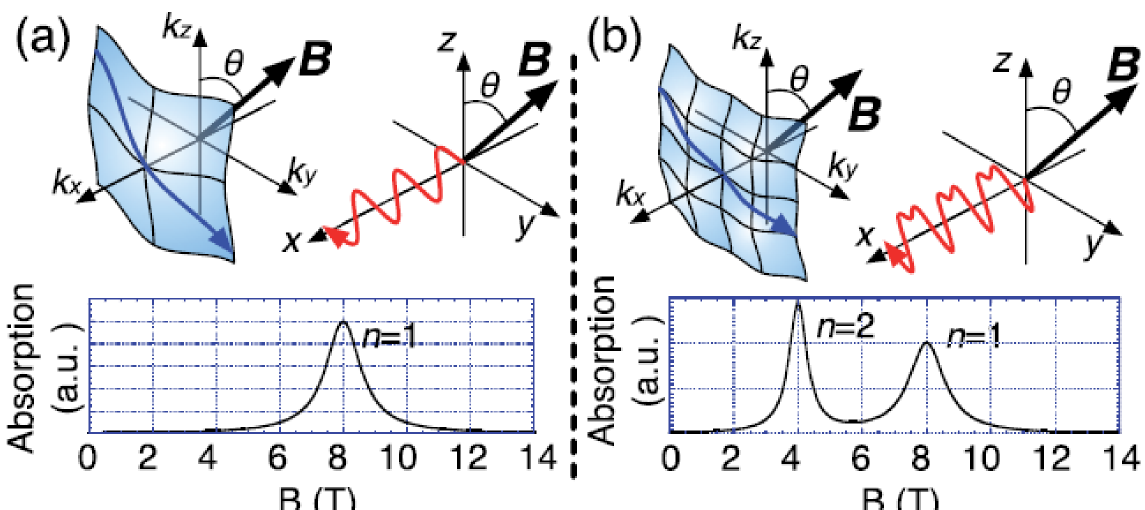
$$\Downarrow$$

$$\frac{\omega}{B\cos\theta} = \left(\frac{ecv_F}{\hbar}\right)\left(p\frac{b}{c} \pm \tan\theta\sin\phi\right)$$







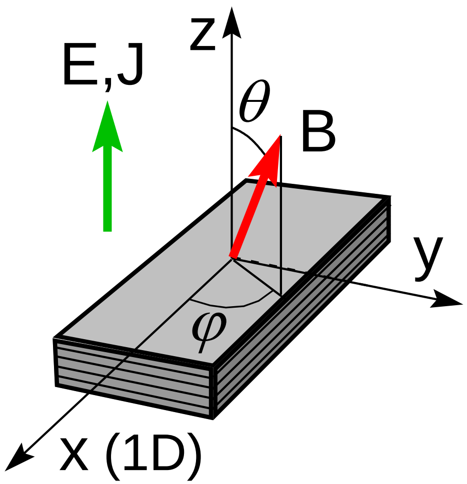


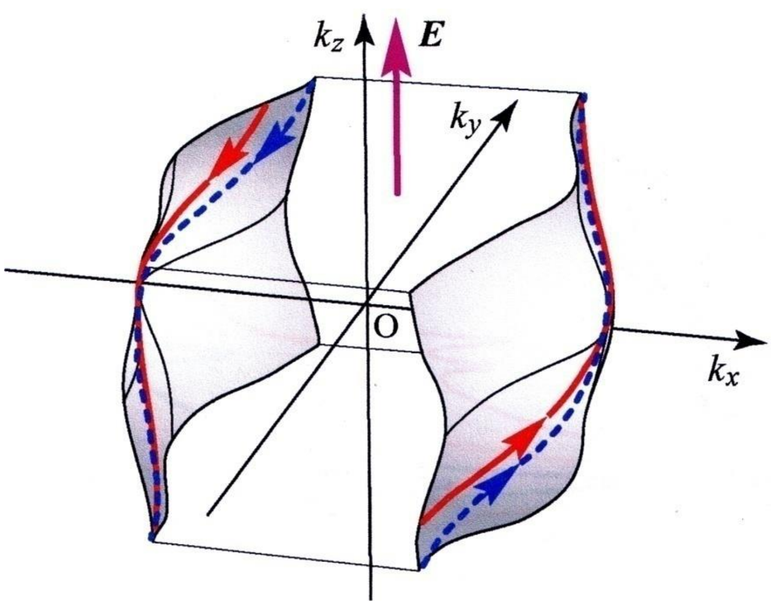
※H. Ohta, M. Kimata, and Y. Oshima, Sci. Technol. Adv. Mater. **10**, 024310 (2009).

Electron Orbital Motion under Magnetic and Electric Fields

※ K. Kobayashi, M. Saito, E. Ohmichi, and T. Osada, Phys. Rev. Lett. **96**, 126601 (2006).

● splitting of Lebed resonance under interlayer electric fields





- model of band dispersion of Q1D conductors

$$E(\mathbf{k}) = \hbar v_F\left(|k_x| - k_F\right) - 2t_b \cos bk_y - 2t_c \cos ck_z$$

- electron orbital motion in **k**-space

$$\begin{cases} \mathbf{v}(\mathbf{k}) = \dfrac{1}{\hbar}\dfrac{\partial E(\mathbf{k})}{\partial \mathbf{k}} \\ \hbar\dot{\mathbf{k}} = (-e)\mathbf{v}(\mathbf{k})\times\mathbf{B} + (-e)\mathbf{E} \approx (-e)\mathbf{v}(\mathbf{k})\times(\mathbf{B}+\mathbf{B}_{\mathrm{eff}}) \end{cases}$$

- effective magnetic field

$$B_{\mathrm{eff}} \equiv \left(0,\ \pm\frac{E_z}{v_{\mathrm{F}}},\ 0\right)$$

- commensurability condition of open orbits

$$\frac{B_y \pm E_z/v_F}{B_z} = p\frac{b}{c} \quad \Longrightarrow \quad \frac{B_y}{B_z} = p\frac{b}{c} \mp \frac{E_z}{v_F B_z}$$

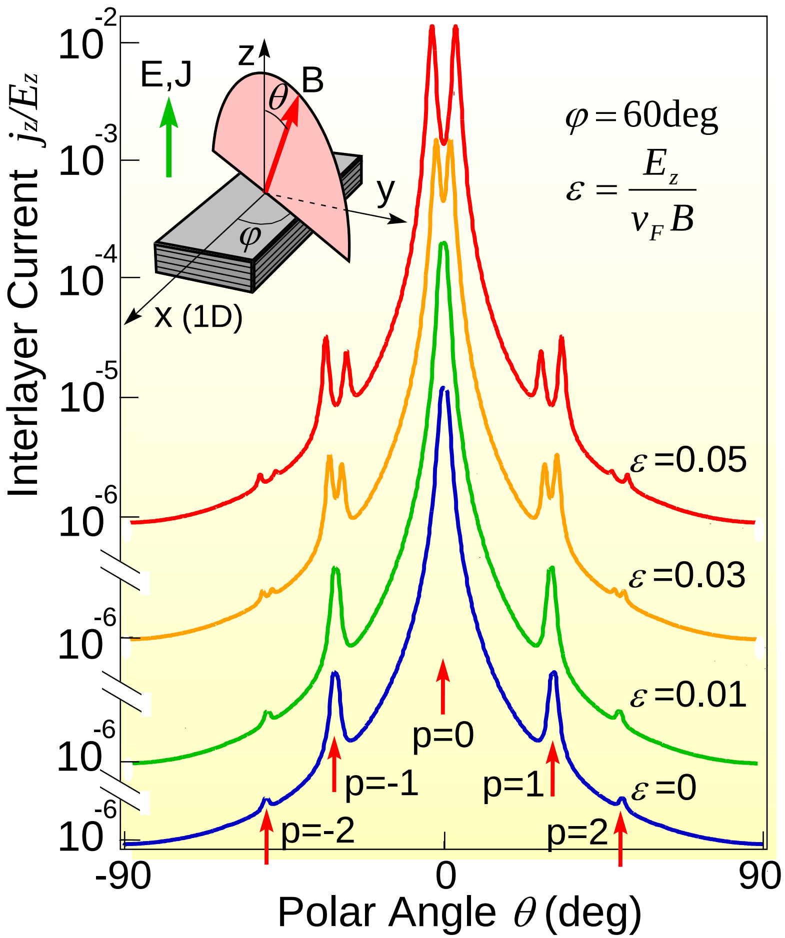
- evaluation of interlayer current

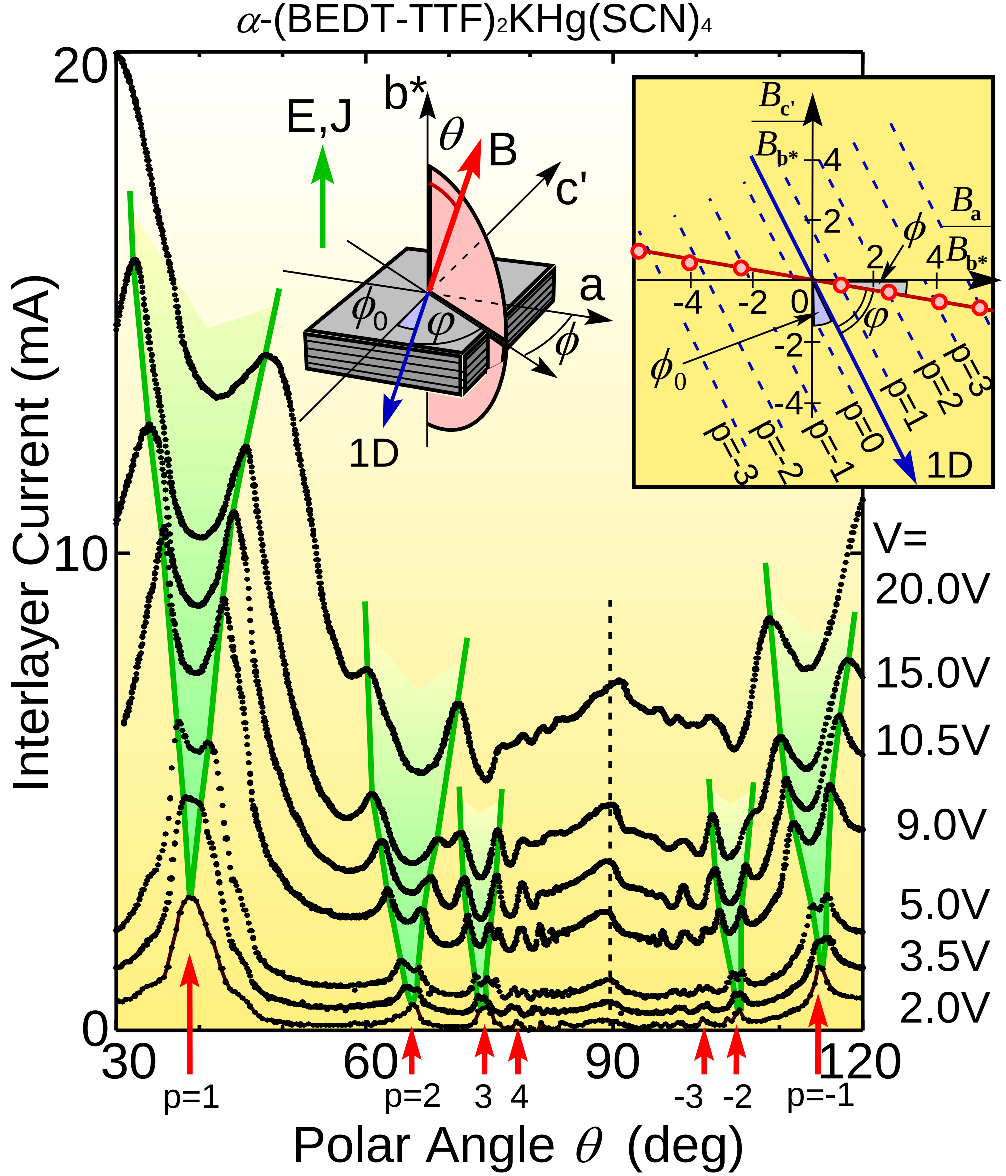
$$\boxed{\frac{j_z}{E_z} \approx N(E_F)\left(\frac{et_c c}{\hbar}\right)^2 \sum_{\pm,\nu} J_\nu\left(\frac{2t_b c}{\hbar v_F}\frac{B_x}{B_z}\right)^2 \frac{\tau}{1+\left\{\omega_B - \left(\nu\omega_z \pm \omega_y\right)\right\}^2\tau^2}}$$

$$N(E_{\mathrm{F}}) \equiv \frac{4}{2\pi bc\hbar v_{\mathrm{F}}},\quad \omega_{\mathrm{B}} \equiv \frac{ceE_z}{\hbar},\quad \omega_z \equiv v_{\mathrm{F}}\frac{beB_z}{\hbar},\quad \omega_y \equiv v_{\mathrm{F}}\frac{ceB_y}{\hbar}$$

Splitting of Lebed Resonance under Interlayer Electric Fields

$$\frac{j_z}{E_z} \approx N(E_F)\left(\frac{e t_c c}{\hbar}\right)^2 \sum_{\pm,\nu} J_\nu\left(\frac{2 t_b c}{\hbar v_F}\frac{B_x}{B_z}\right)^2 \frac{\tau}{1+\left\{\omega_B-\left(\nu\omega_z \pm \omega_y\right)\right\}^2 \tau^2}$$

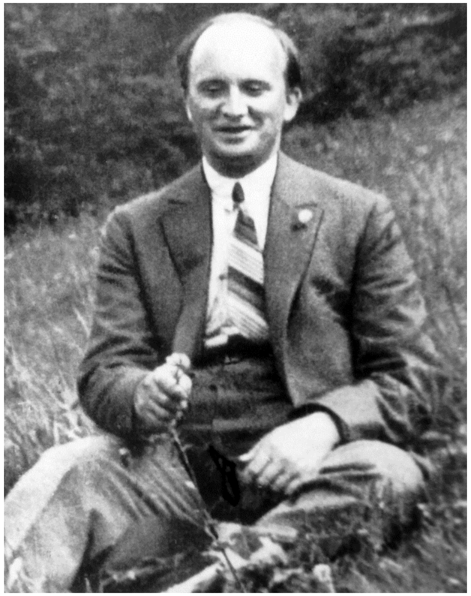




2. 量子振動とFermi面

§ 2. Quantum Oscillations

0. Landau levels
1. Lifshitz-Kosevich formula
2. analysis of Shubnikov-de Haas effect
3. Fourier analysis: FFT & MEM
4. magnetic breakdown effect
5. quantum interference effect
6. Berry phase



Lev Vasil'evich Shubnikov

Semiclassical Picture of Landau Quantization

- equation of motion of an electron wave packet in the **k**-space

$$\begin{cases} \mathbf{v} \equiv \dot{\mathbf{R}} = \dfrac{1}{\hbar}\dfrac{\partial E(\mathbf{k})}{\partial \mathbf{k}} \\ \hbar\dot{\mathbf{k}} = -e\mathbf{v}\times\mathbf{B} \end{cases}$$

group velocity is normal to the equi-energy surface

$$\begin{aligned} \hbar\dot{\mathbf{k}}\times\mathbf{e}_\mathbf{B} &= (-e\mathbf{v}\times\mathbf{B})\times\mathbf{e}_\mathbf{B} \\ &= -e(\mathbf{v}\cdot\mathbf{e}_\mathbf{B})\mathbf{B} + e(\mathbf{B}\cdot\mathbf{e}_\mathbf{B})\mathbf{v} \\ &= -ev_\mathbf{B}\mathbf{B} + eB\dot{\mathbf{R}} \end{aligned} \Rightarrow \mathbf{v}_\perp = \dot{\mathbf{R}}_\perp = \frac{\hbar}{eB}\dot{\mathbf{k}}\times\mathbf{e}_\mathbf{B} = l^2\dot{\mathbf{k}}\times\mathbf{e}_\mathbf{B}$$

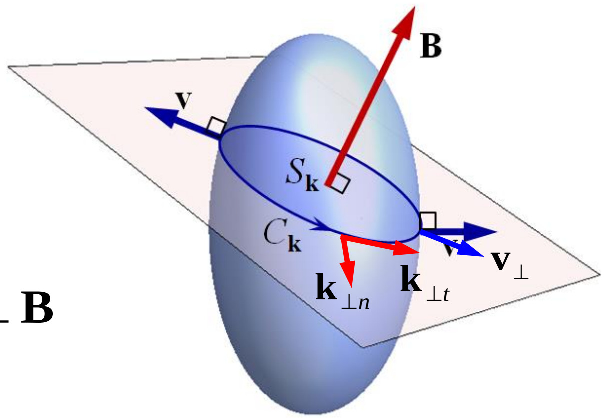
$$\Downarrow$$

$$\hbar\dot{\mathbf{k}} = -\frac{e}{\hbar}\frac{\partial E(\mathbf{k})}{\partial \mathbf{k}}\times\mathbf{B}$$

$$\Downarrow$$

① on an equi-energy surface: $\dot{\mathbf{k}} \perp \dfrac{\partial E(\mathbf{k})}{\partial \mathbf{k}}$

② on a plane perpendicular to the magnetic field: $\dot{\mathbf{k}} \perp \mathbf{B}$



- semiclassical orbital quantization

$$\oint \mathbf{P}\cdot d\mathbf{R} = (n+\gamma)h$$ (Bohr-Sommerfeld condition)

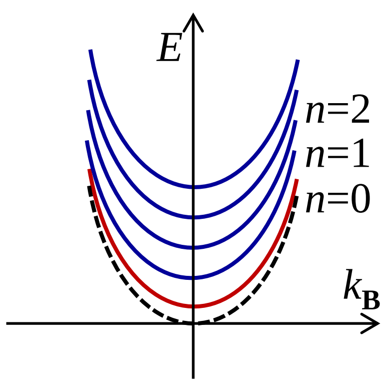
$$d\mathbf{R}_\perp = \frac{\hbar}{eB}d\mathbf{k}\times\mathbf{e}_\mathbf{B}$$

$\mathbf{R}_\perp$は磁場に垂直な平面上への$\mathbf{R}$の射影ベクトル

$$\begin{aligned} \oint \mathbf{P}\cdot d\mathbf{R}_\perp &= \oint \hbar\mathbf{K}\cdot d\mathbf{R}_\perp = \oint(\hbar\mathbf{k} - e\mathbf{A}(\mathbf{R}))\cdot d\mathbf{R}_\perp \\ &= \hbar l^2\oint \mathbf{k}\cdot(d\mathbf{k}\times\mathbf{e}_\mathbf{B}) - e\iint_{S_{\mathbf{R}\perp}}(\nabla\times\mathbf{A})\cdot d\mathbf{S}_{\mathbf{R}\perp} \\ &= \hbar l^2\mathbf{e}_\mathbf{B}\cdot\oint(\mathbf{k}\times d\mathbf{k}) - e\iint_{S_{\mathbf{R}\perp}}\mathbf{B}\cdot d\mathbf{S}_{\mathbf{R}\perp} \\ &= \hbar l^2\times 2\mathbf{S}_\mathbf{k} - eB\mathbf{S}_{\mathbf{R}\perp} = \hbar l^2\mathbf{S}_\mathbf{k} \end{aligned}$$

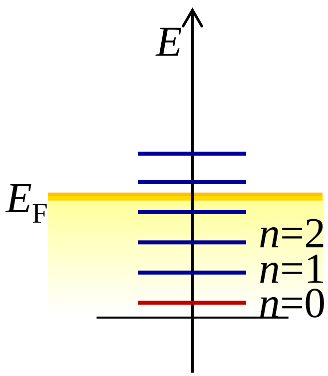
$$\Downarrow$$

$$\boxed{S_\mathbf{k} = \frac{2\pi}{l^2}(n+\gamma)}$$ (Onsager-Lifshits rule)

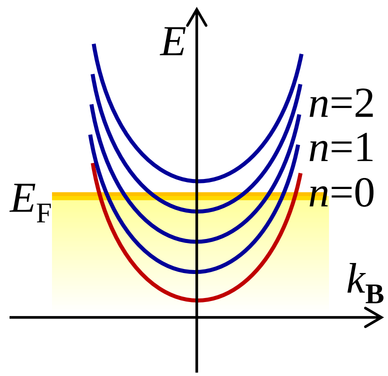


●Landau levels

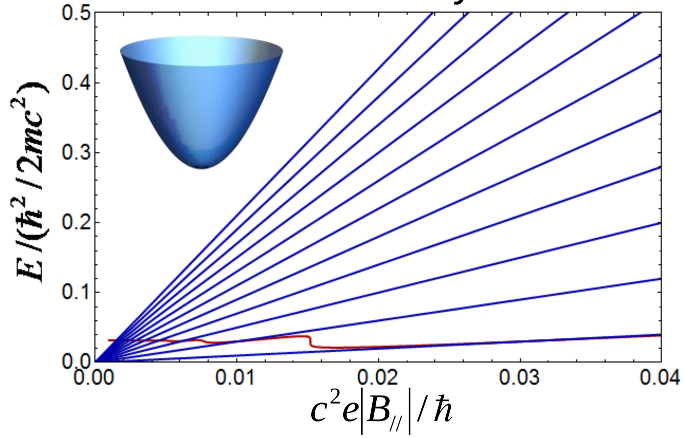
2D system



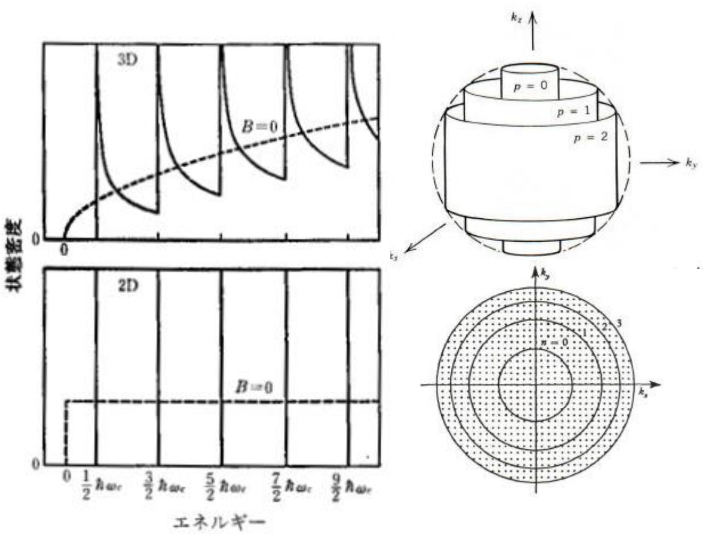
3D system



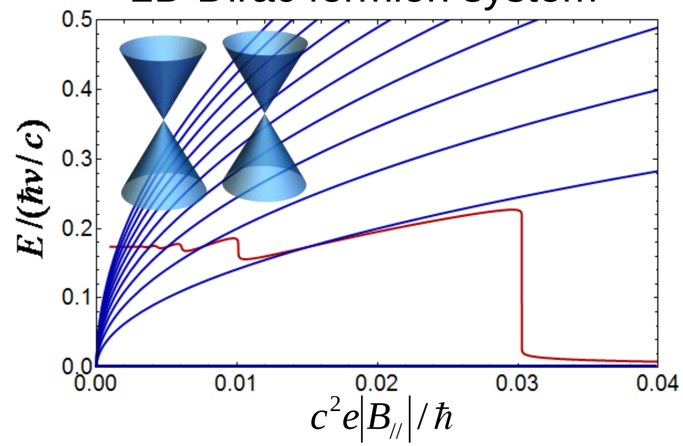
2D electron system



●density of states of Landau levels



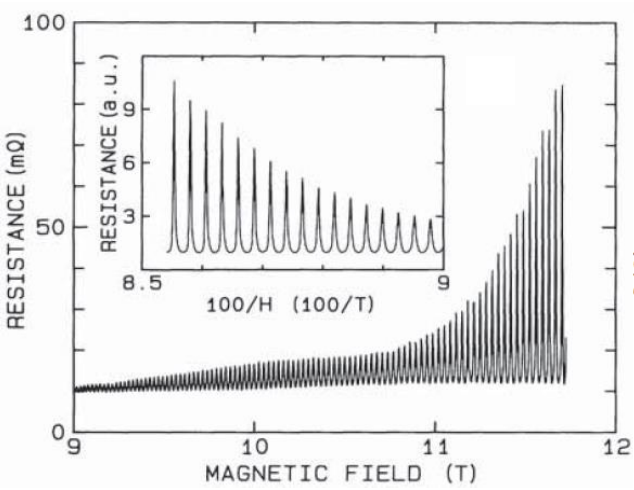
2D Dirac fermion system



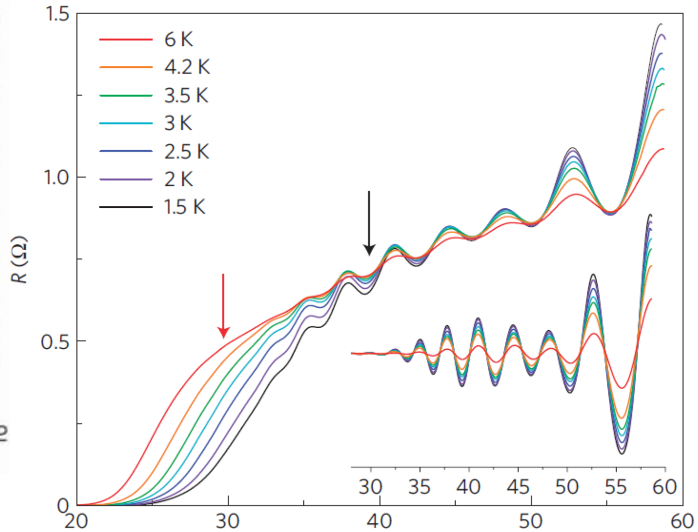
Shubnikov-de Haas Effect in Magnetotransport

● Shubnikov-de Haas oscillation

layered organic conductor:
β_H- (BEDT-TTF)$_2$I$_3$

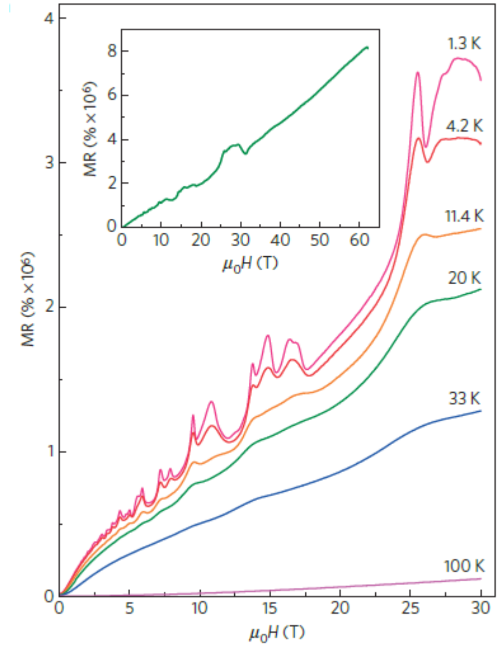


vortex liquid phase
in YBa$_2$Cu$_3$O$_{6.59}$



※B. J. Ramshaw et al., Nat. Phys. **7**, 234 (2011).

Weyl semimetal NbP



※Chandra Shekhar, et al., Nat. Phys. **11**, 645 (2015).

Lifshitz-Kosevich Formula for Shubnikov-de Haas Effect

- density of states (in the case of 3D conductor)

$$D(E)=\frac{1}{2\pi}\left(\frac{2m^*}{\hbar^2}\right)^{1/2}\sum_{N,\sigma_z}\frac{1}{2\pi l^2}\Big/\sqrt{E-\hbar\omega_c\left(N+\frac{1}{2}\right)-\frac{1}{2}g\mu_B\sigma_z B} \longleftarrow \sum_{N=0}^{\infty} f\left(N+\frac{1}{2}\right)=\int_0^\infty f(x)dx+2\sum_{r=1}^{\infty}(-1)^r\int_0^\infty f(x)\cos(2\pi rx)dx$$

$$\approx D_0(E)\left\{1+\sum_{r=1}^{\infty}(-1)^r\sqrt{\frac{\hbar\omega_c}{2Er}}\cos\left(\frac{2\pi E}{\hbar\omega_c}r-\frac{\pi}{4}\right)\cos\left(\pi\frac{g\mu_B B}{\hbar\omega_c}r\right)\right\}$$

- magnetoresistance (in the case of 3D conductor)

longitudinal magnetoresistance: $\rho_{//}=\rho_{zz}=\dfrac{1}{\sigma_{zz}}\cong\rho_0\left\{1+\sum_{r=1}^{\infty}b_r\cos\left(\dfrac{2\pi\mu}{\hbar\omega_c}r-\dfrac{\pi}{4}\right)\right\}$

transverse magnetoresistance: $\rho_{\perp}=\rho_{xx}=\rho_0\left\{1+\dfrac{5}{2}\sum_{r=1}^{\infty}b_r\cos\left(\dfrac{2\pi\mu}{\hbar\omega_c}r-\dfrac{\pi}{4}\right)+R\right\}$

$$b_r=(-1)^r\sqrt{\frac{\hbar\omega_c}{2\mu r}}\,\frac{2\pi^2 rk_BT/\hbar\omega_c}{\sinh(2\pi^2 rk_BT/\hbar\omega_c)}\cos\left(\pi\frac{g\mu_B B}{\hbar\omega_c}r\right)e^{-2\pi\Gamma r/\hbar\omega_c}$$

overlap of DOS of
Landau subband (3D)

interference factor
due to spin splitting

$$R=\frac{3}{4}\frac{\hbar\omega_c}{2\mu}\left\{\sum_{r=1}^{\infty}b_r\left[\alpha_r\cos\left(\frac{2\pi\mu}{\hbar\omega_c}r\right)+\beta_r\sin\left(\frac{2\pi\mu}{\hbar\omega_c}r\right)\right]-\ln\left(1-e^{4\pi\Gamma/\hbar\omega_c}\right)\right\}$$

$$\alpha_r=2\sqrt{r}\sum_{s=1}^{\infty}\frac{1}{\sqrt{s(r+s)}}e^{-4\pi s\Gamma/\hbar\omega_c},\quad \beta_r=\sqrt{r}\sum_{s=1}^{r-1}\frac{1}{\sqrt{s(r-s)}}$$

※R can be neglected in sinusoidal oscillations（r=1）.

Data Analysis of Shubnikov-de Haas Oscillations

$$\frac{\Delta\rho}{\rho_0} \propto \left[\sqrt{B}\,\frac{\chi}{\sinh\chi}\,e^{-\chi_D}\cos\left(\pi\frac{g\mu_B B}{\hbar\omega_c}\right)\right]\cdot\cos\left(l^2 S_{\mathbf{k}} + \phi\right)$$ for large N (not quantum limit)

基本波（r=1）成分の振幅b_1：正値、3次元系の場合

$$\chi = \frac{2\pi^2 k_B T}{\hbar\omega_c}$$

$$\chi_D = \frac{2\pi^2 k_B T_D}{\hbar\omega_c} = \frac{2\pi^2\Gamma}{\hbar\omega_c} = \frac{\pi}{\omega_c\tau}$$

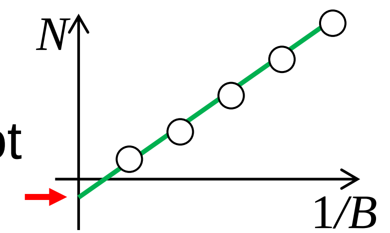
$$\phi = \delta - 2\pi\gamma = \begin{cases} +\dfrac{\pi}{4} - 2\pi\left(\dfrac{1}{2} - \dfrac{\Phi}{2\pi}\right) & \text{(minimum cross section)} \\ -\dfrac{\pi}{4} - 2\pi\left(\dfrac{1}{2} - \dfrac{\Phi}{2\pi}\right) & \text{(maximum cross section)} \\ 0 - 2\pi\left(\dfrac{1}{2} - \dfrac{\Phi}{2\pi}\right) & \text{(2D Fermi surface)} \end{cases}$$

$$\Phi = \begin{cases} 0 & \text{(normal)} \\ 1 & \text{(Dirac)} \end{cases}$$

- period ⇒ extremal cross section of Fermi surface

$$\tilde{S}_{\mathbf{k}}(E) = \frac{2\pi eB}{\hbar}\left(N + \frac{1}{2} - \frac{\gamma_n(\tilde{C}_{\mathbf{k}})}{2\pi}\right) \Rightarrow \Delta\left(\frac{1}{B}\right) = \frac{2\pi e}{\hbar\tilde{S}_{\mathbf{k}}}$$

Landau plot



抵抗ピークの位置

- N-intercept ⇒ 1/2 (ordinary) or 0 (Dirac) in 2D

- amplitude of fundamental component (r=1): b_1

temperature dependence: $\dfrac{\chi}{\sinh\chi}$

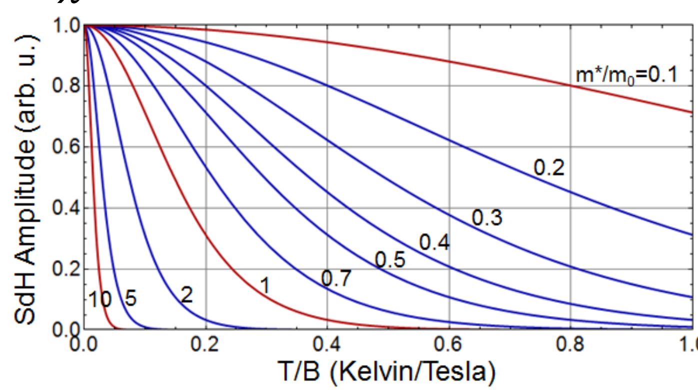
⇒ m^* : effective mass

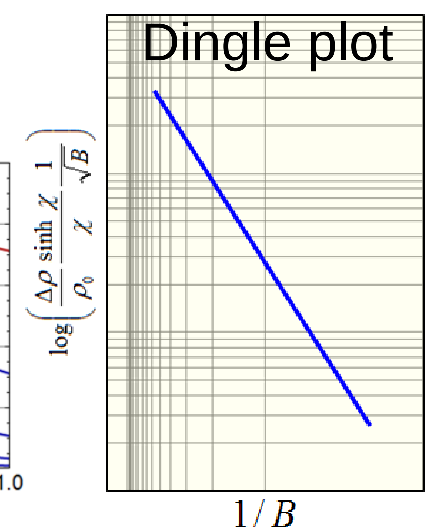
$$\chi = \frac{2\pi^2 k_B T}{\hbar\omega_c} = 14.7[\mathrm{T/K}]\frac{m^*}{m_0}\frac{T}{B}$$

magnetic field dependence

⇒ T_D : Dingle temperature

$$\Gamma = k_B T_D \qquad \frac{\hbar}{\tau} = 2\pi\Gamma$$

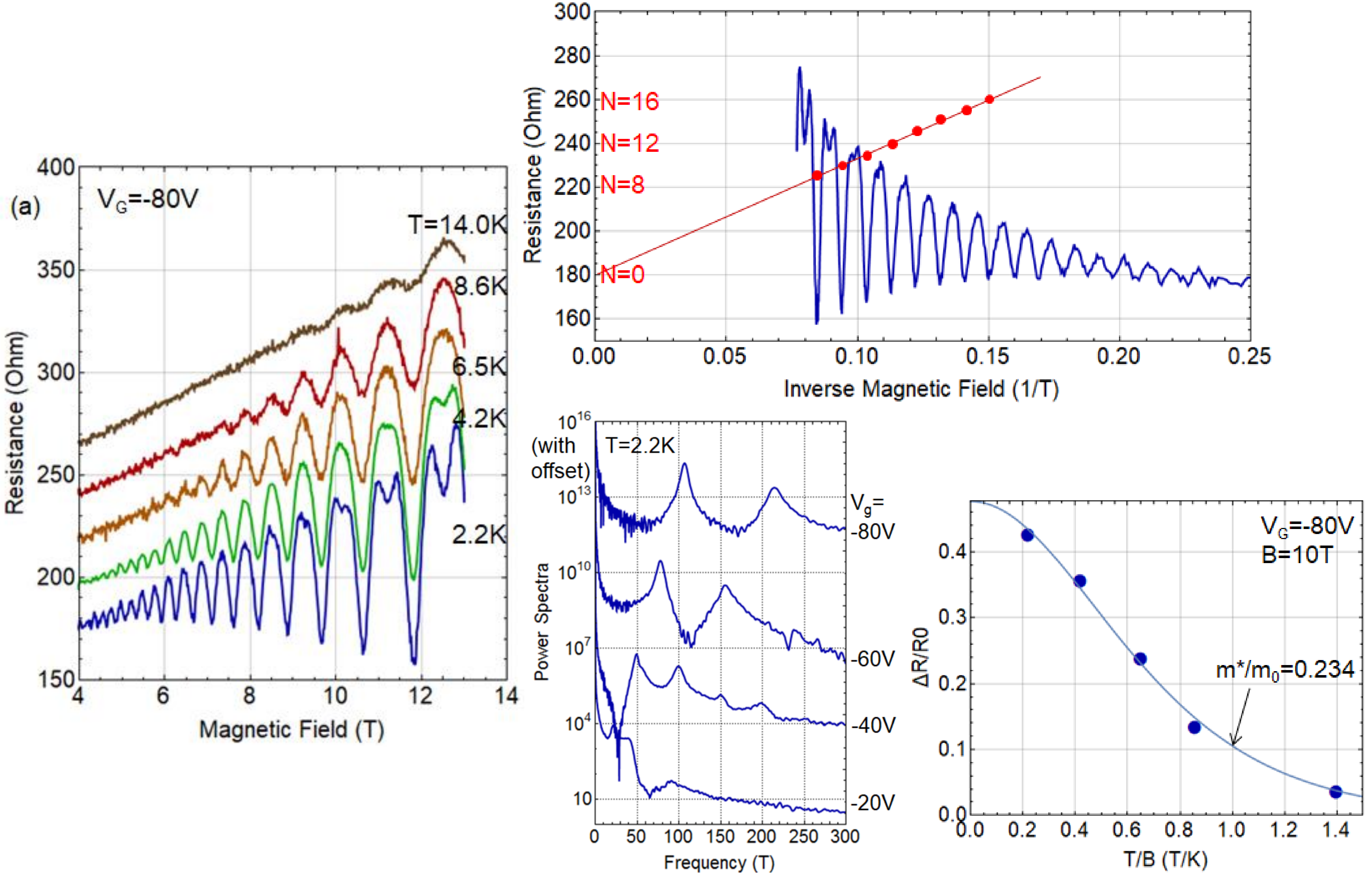




Example for Data Analysis of SdH Oscillations

● thin-film black phosphorus FET

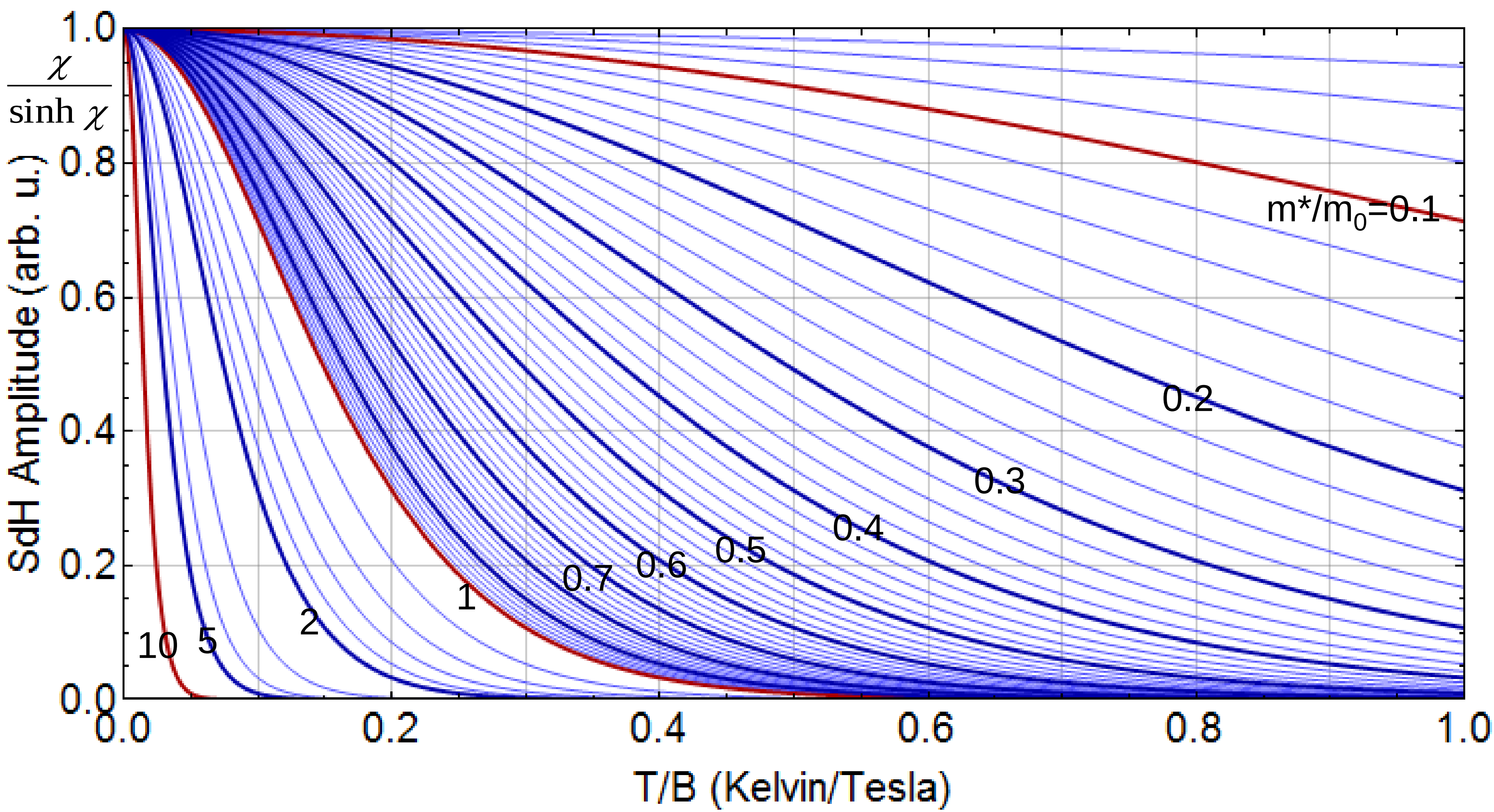
※K. Hirose, T. Osada, K. Uchida, T. Taen, K. Watanabe, T. Taniguchi, and Y. Akahama, Appl. Phys. Lett. **113**, 193101 (2018).



Temperature Dependence of Shubnikov-de Haas Amplitude

- temperature dependence of fundamental amplitude at a fixed magnetic field

$$\frac{\Delta\rho}{\rho_0} \propto \frac{\chi}{\sinh\chi} \qquad \chi \equiv \frac{2\pi^2 k_B T}{\hbar\omega_c} = 14.7[\mathrm{T/K}]\frac{m^*}{m_0}\frac{T}{B}$$



Temperature Dependence of Shubnikov-de Haas Amplitude

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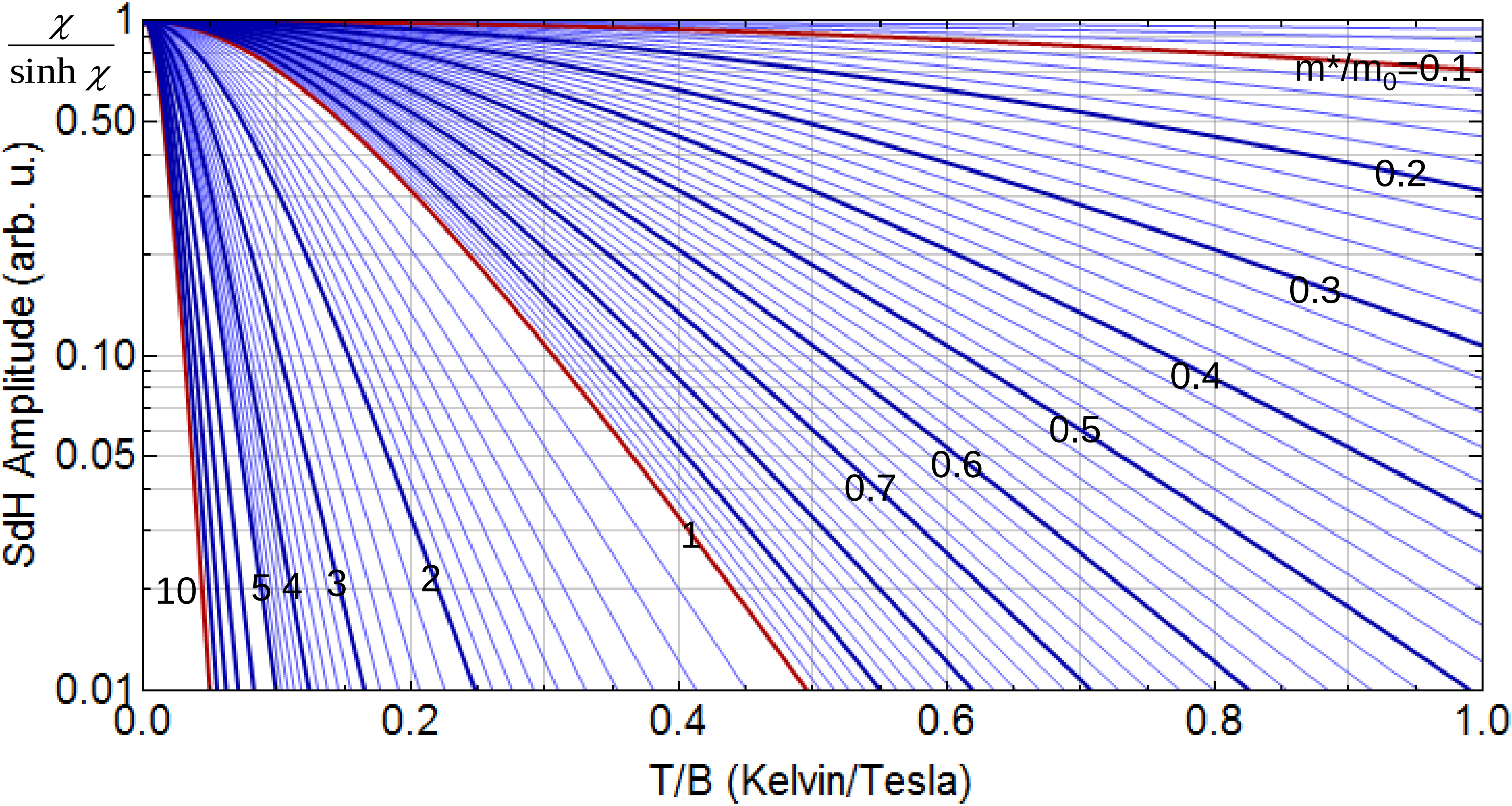


Discrete Fourier Transform and Fast Fourier Transform (FFT)

- Fourier transform in $(-\infty,\infty)$

$$\begin{cases} x(t) = \int_{-\infty}^{\infty} X(\omega) e^{+i\omega t} d\omega \\ X(\omega) = \dfrac{1}{2\pi} \int_{-\infty}^{\infty} x(t) e^{-i\omega t} dt \end{cases}$$

- discrete Fourier transform (DFT) in $[0, T)=[0, N\Delta t)$

$$\begin{cases} x_j = x(t_j) = \sum_{k=0}^{N-1} X_k e^{+i\omega_k t_j} = \sum_{k=0}^{N-1} X_k e^{+i\left(\frac{2\pi}{T}k\right)\left(\frac{T}{N}j\right)} \\ X_k = X(\omega_k) = \dfrac{1}{N} \sum_{j=0}^{N-1} x_j e^{-i\omega_k t_j} = \dfrac{1}{N} \sum_{j=0}^{N-1} x_j e^{-i\left(\frac{2\pi}{T}k\right)\left(\frac{T}{N}j\right)} \end{cases}$$

$$\begin{pmatrix} x_0 \\ x_1 \\ \vdots \\ x_{N-1} \end{pmatrix} = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ 1 & W & \cdots & W^{N-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & W^{N-1} & \cdots & W^{(N-1)^2} \end{pmatrix} \begin{pmatrix} X_0 \\ X_1 \\ \vdots \\ X_{N-1} \end{pmatrix}$$

$$W \equiv e^{+i\left(\frac{2\pi}{T}\right)\left(\frac{T}{N}\right)} = e^{+i\frac{2\pi}{N}}$$

$$W^{j\cdot k} = e^{+i\left(\frac{2\pi}{T}k\right)\left(\frac{T}{N}j\right)} = \left(e^{+i\frac{2\pi}{N}}\right)^{j\cdot k}$$:phase rotation factor

- fast Fourier transform (FFT) in $[0, T)=[0, 2^p\Delta t)$ ※ Cooley & Tukey (1965)

高速フーリエ変換

factorization of $W^{j\cdot k}$ using periodicity of $e^{i(2\pi/2^p)}$ ⇒ "butterfly" calculation

因数分解

Maximum Entropy Method (MEM) = Auto-Regressive Model

最大エントロピー法　　自己回帰モデル

- fitting to linear combination of M damped oscillators ($r_k>0$) ($j=1, \dots , N$)

$$x_j = x(t_j) = \sum_{k=1}^{M} A_k e^{(-r_k + i\omega_k)t_j} = \sum_{k=1}^{M} A_k \left[e^{(-r_k + i\omega_k)\Delta t} \right]^j = \sum_{k=1}^{M} A_k z_k{}^j$$

solution

- auto-regressive (AR) model

自己回帰モデル

$$x_j = \sum_{k=1}^{M} a_k x_{j-k} = a_1 x_{j-1} + a_2 x_{j-2} + \cdots + a_M x_{j-M}$$

: recurrence relation (difference eq.)

漸化式

$$\mathbf{d} = \begin{pmatrix} x_M \\ x_{M+1} \\ \vdots \\ x_{N-1} \end{pmatrix} = \begin{pmatrix} x_{M-1} & x_{M-2} & \cdots & x_0 \\ x_{M+0} & x_{M-1} & \cdots & x_1 \\ \vdots & \vdots & \ddots & \vdots \\ x_{N-2} & x_{N-3} & \cdots & x_{N-1-M} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_M \end{pmatrix} \equiv [H]\mathbf{a}$$

least squares method

$$\mathbf{a} = \left([H]^t [H] \right)^{-1} [H]^t \mathbf{d}$$

- characteristic equation

特性方程式

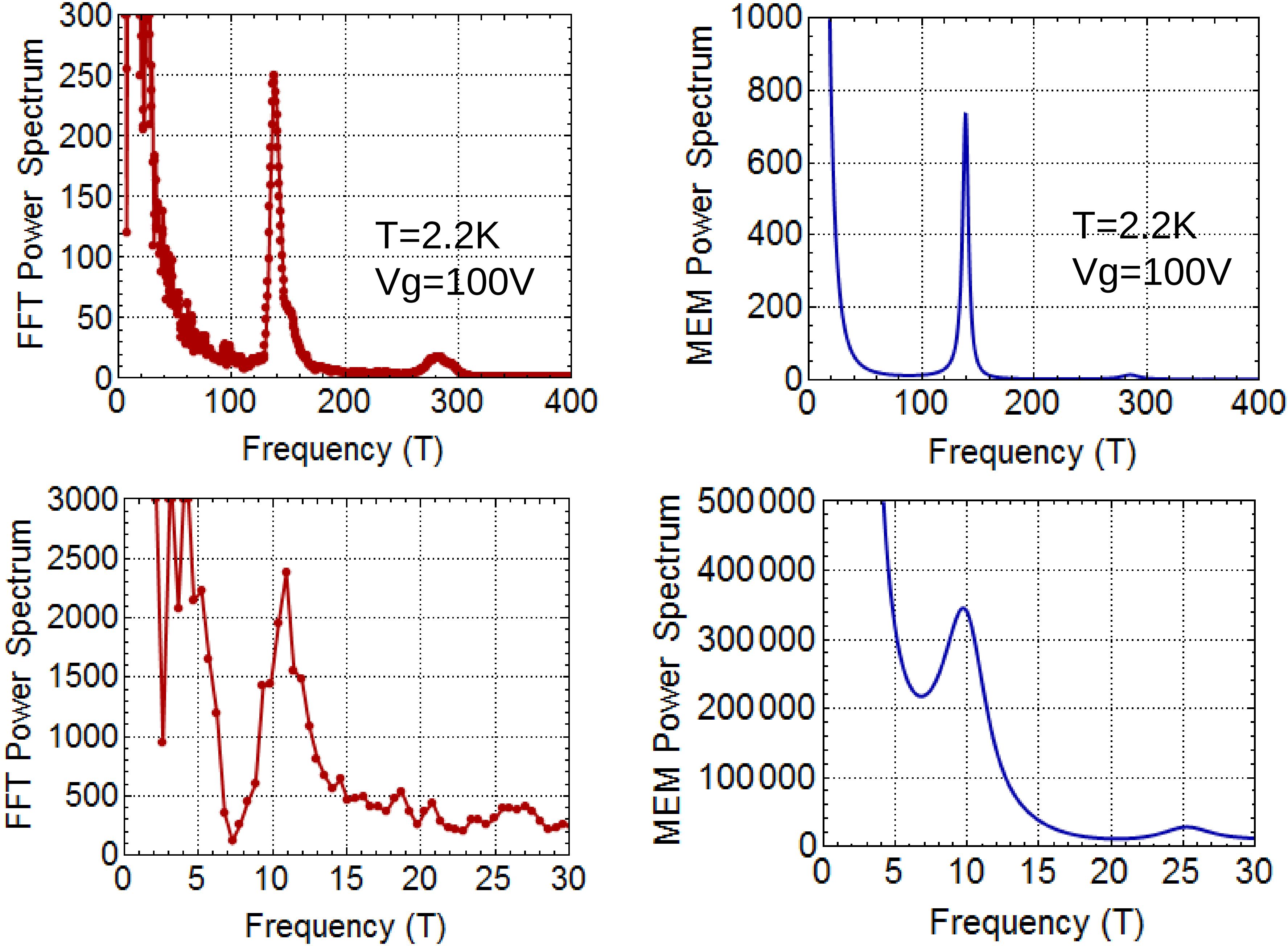
$$z^M = \sum_{k=1}^{M} a_k z^{M-k} = a_1 z^{M-1} + a_2 z^{M-2} + \cdots + a_M z^0$$

roots ($k=1,\dots,M$): $z_k = e^{(-r_k \pm i\omega_k)\Delta t}$

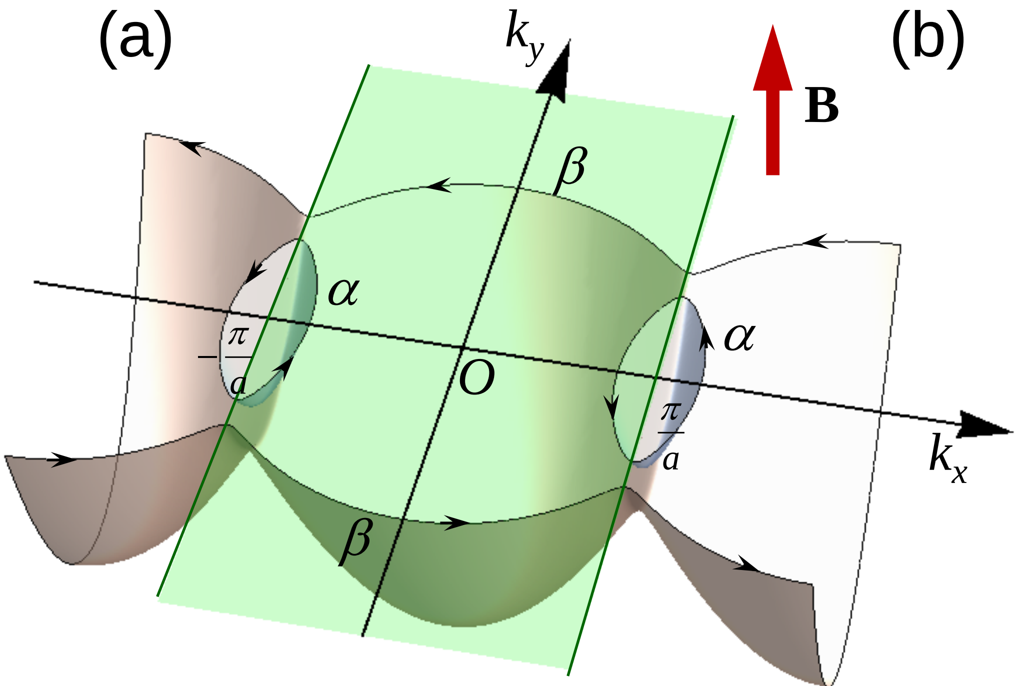
- power spectral density in AR model

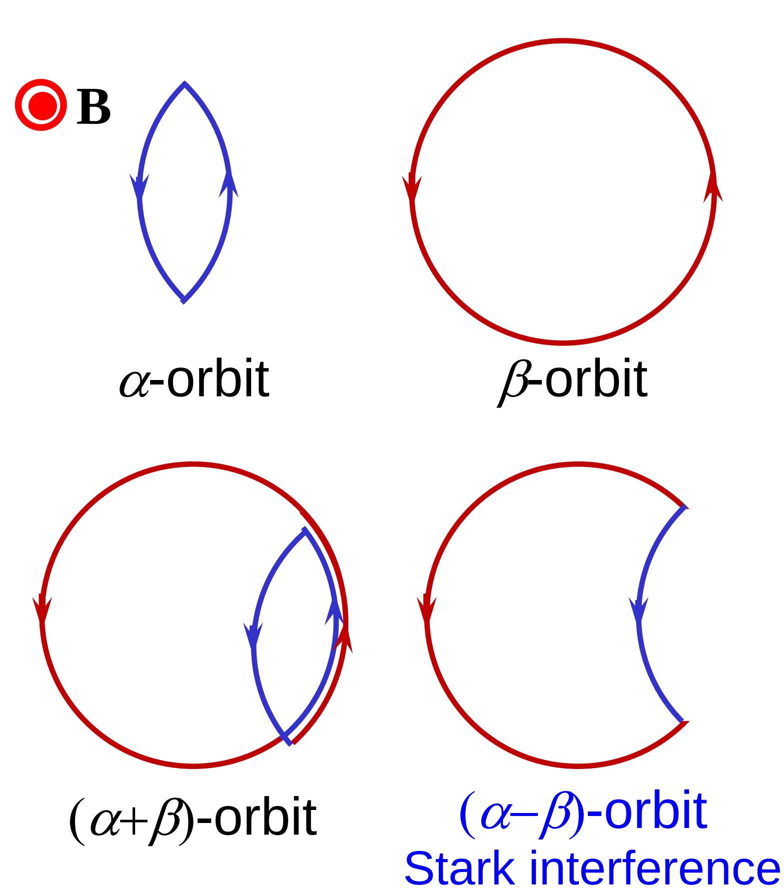
$$P(\omega) = \frac{1}{\left| 1 - \sum_{k=1}^{M} a_k (e^{i\omega\Delta t})^{-k} \right|^2}$$

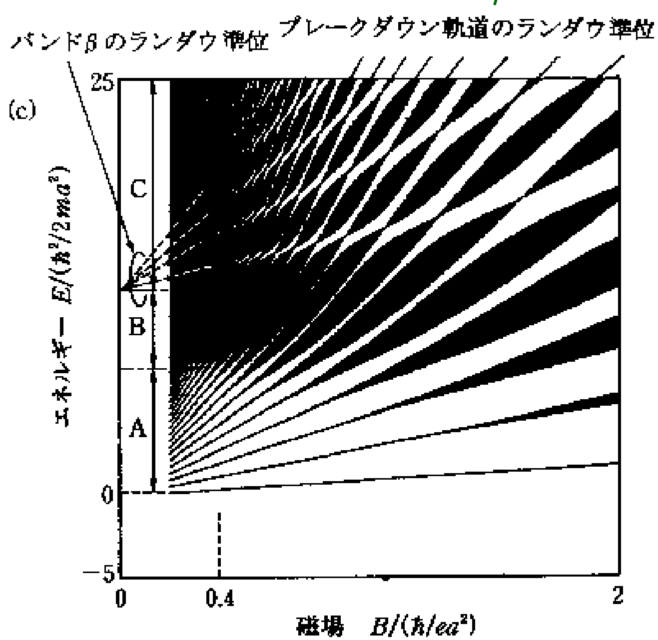
FFT and MEM Spectra of SdH Oscillations of 2D Electrons in BP



Quantum Oscillations in Magnetic Breakdown Systems







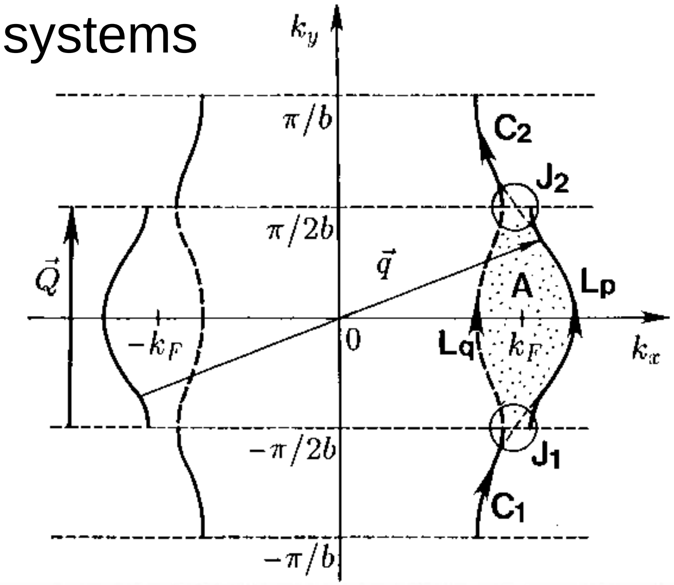
$$\boxed{\hat{\mathbf{k}} \times \hat{\mathbf{k}} = -i\frac{e\mathbf{B}(\hat{\mathbf{R}})}{\hbar}}$$

$$\Delta k_x \cdot \Delta k_y \geq 2\pi \frac{eB_z}{\hbar} = S_{BZ} \cdot \frac{\Phi_z}{\Phi_0}$$

$$\boxed{[\hat{k}_x, \hat{k}_y] = -i\frac{eB_z}{\hbar}}$$

"Stark Quantum Interferometer": Aharonov-Bohm effect in k-space

- Stark interference effect in magnetic breakdown systems



- phase difference of real space orbits

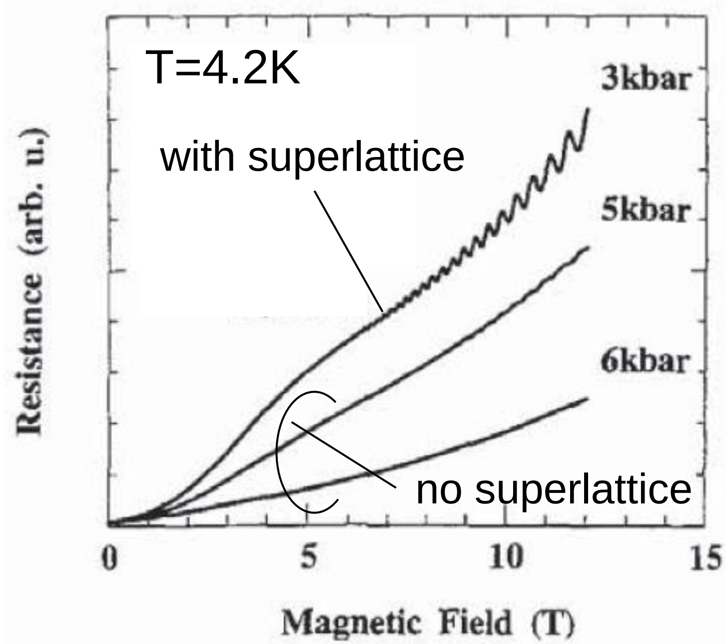
$$\begin{aligned}\theta_{L_p}-\theta_{L_q}&=-\frac{e}{\hbar}\int_{L_p}\mathbf{A}\cdot d\mathbf{l}+\frac{e}{\hbar}\int_{L_q}\mathbf{A}\cdot d\mathbf{l}\\&=\frac{e}{\hbar}\oint_{L_q-L_p}\mathbf{A}\cdot d\mathbf{l}=\frac{e}{\hbar}\iint(\nabla\times\mathbf{A})\cdot d\mathbf{S}\\&=\frac{e}{\hbar}\iint\mathbf{B}\cdot d\mathbf{S}=\frac{e}{\hbar}BS\\&=\frac{S}{l^2}=\frac{(S_\mathbf{k}l^4)}{l^2}=S_\mathbf{k}l^2=\frac{\hbar}{eB}S_\mathbf{k}\end{aligned}$$

- period of interference effect

$$\Delta\left(\frac{1}{B}\right)=\frac{2\pi e}{\hbar S_\mathbf{k}}$$

※ Stark quantum interference in $(TMTSF)_2ClO_4$

superlattice potential : $H'=V\cos\dfrac{\pi}{b}y$
(P < 4.0 kbar)



H. Shinagawa, S. Kagoshima, T. Osada, and N. Miura, Physica **B201**, 490-492 (1994).

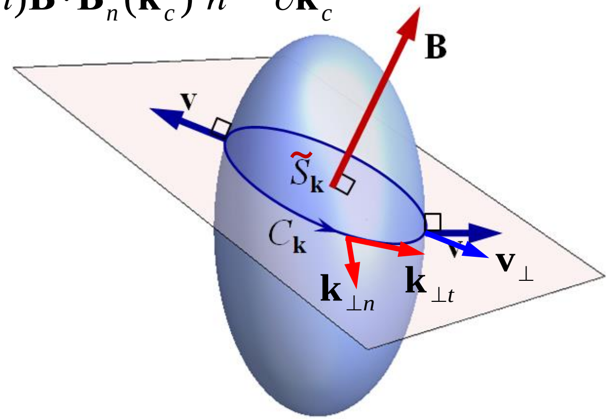
Berry Curvature Effect on Landau Quantization

- wave packet motion in k-space with Berry curvature under magnetic field

$$\begin{cases} \dot{\mathbf{r}}_c = \dfrac{1}{\hbar}\dfrac{\partial \tilde{E}_n(\mathbf{k}_c)}{\partial \mathbf{k}_c} - \dot{\mathbf{k}}_c \times \mathbf{B}_n(\mathbf{k}_c) \\ \hbar\dot{\mathbf{k}}_c = -e\dot{\mathbf{r}}_c \times \mathbf{B} \end{cases}$$

$$\Rightarrow \quad \hbar\dot{\mathbf{k}}_c = -e\frac{1}{1+(e/\hbar)\mathbf{B}\cdot\mathbf{B}_n(\mathbf{k}_c)}\frac{1}{\hbar}\frac{\partial \tilde{E}_n(\mathbf{k}_c)}{\partial \mathbf{k}_c}\times\mathbf{B}$$

$$\dot{\mathbf{k}}_c \perp \frac{\partial \tilde{E}_n(\mathbf{k}_c)}{\partial \mathbf{k}_c}, \quad \dot{\mathbf{k}}_c \perp \mathbf{B}$$



$$\boxed{\tilde{E}_n(\mathbf{k}) \equiv E_n(\mathbf{k}) - \mathbf{B}\cdot\mathbf{m}(\mathbf{k})} \qquad \mathbf{m}(\mathbf{k}) = -i\frac{e}{2\hbar}\langle \nabla_{\mathbf{k}} u_{n\mathbf{k}} | \times [\hat{H}(\mathbf{k}) - E_n(\mathbf{k})] | \nabla_{\mathbf{k}} u_{n\mathbf{k}} \rangle$$

① motion on an equi-energy surface
② motion on a plane perpendicular to the magnetic field

the same k-space orbit as effective mass approximation
but the k-space velocity is modified.

- Landau quantization

- Bohr-Sommerfeld quantization:

$$\oint \left[\{\hbar\mathbf{k}_c - e\mathbf{A}(\mathbf{r}_c)\}\cdot d\mathbf{r}_{c\perp} + \hbar\mathbf{A}_n(\mathbf{k}_c)\cdot d\mathbf{k}_c\right] = \hbar l^2 \tilde{S}_{\mathbf{k}} + \frac{h}{2\pi}\gamma_n(C_{\mathbf{k}}) = h\left(N + \frac{1}{2}\right)$$

- Onsager-Lifshitz quantization rule:

$$\boxed{\tilde{S}_{\mathbf{k}} = \frac{2\pi}{l^2}\left(N + \frac{1}{2} - \frac{\gamma_n(C_{\mathbf{k}})}{2\pi}\right)}$$

$$\mathbf{A}_n(\mathbf{k}) = i\langle u_{n\mathbf{k}} | \nabla_{\mathbf{k}} u_{n\mathbf{k}} \rangle$$

$$\gamma_n(C_{\mathbf{k}}) = \oint_{C_{\mathbf{k}}} \mathbf{A}_n(\mathbf{k})\cdot d\mathbf{k} = \iint_S \mathbf{B}_n(\mathbf{k})\cdot dS_{\mathbf{k}}$$

Phase Analysis of Quantum Oscillations

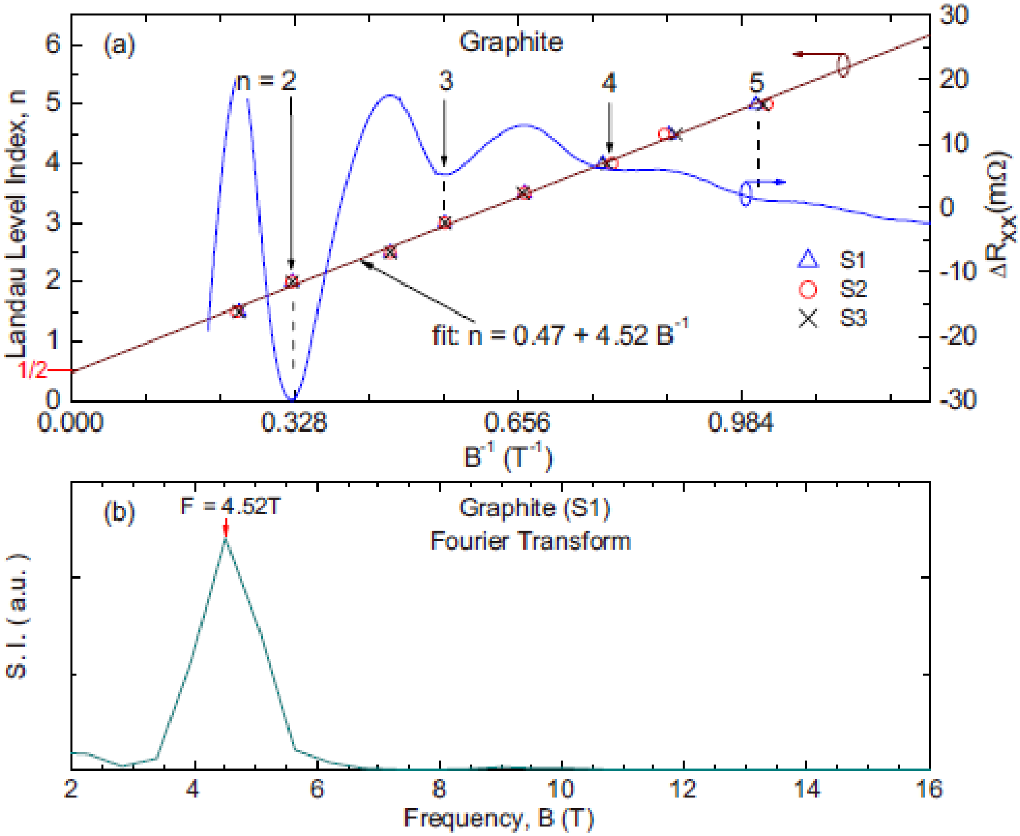
● Landau plot

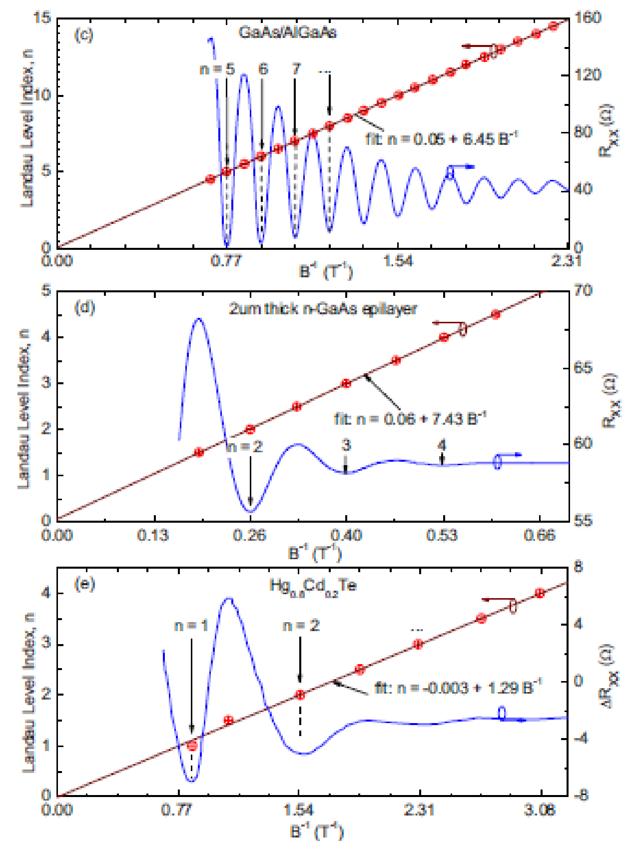
Berry phase → shift of y-intercept from N=0

（space inversion symmetry）

"Transport study of the Berry phase, resistivity rule, and quantum Hall effect in graphite", A. N. Ramanayaka and R. G. Mani, Phys. Rev. B **82**,165327 (2010).

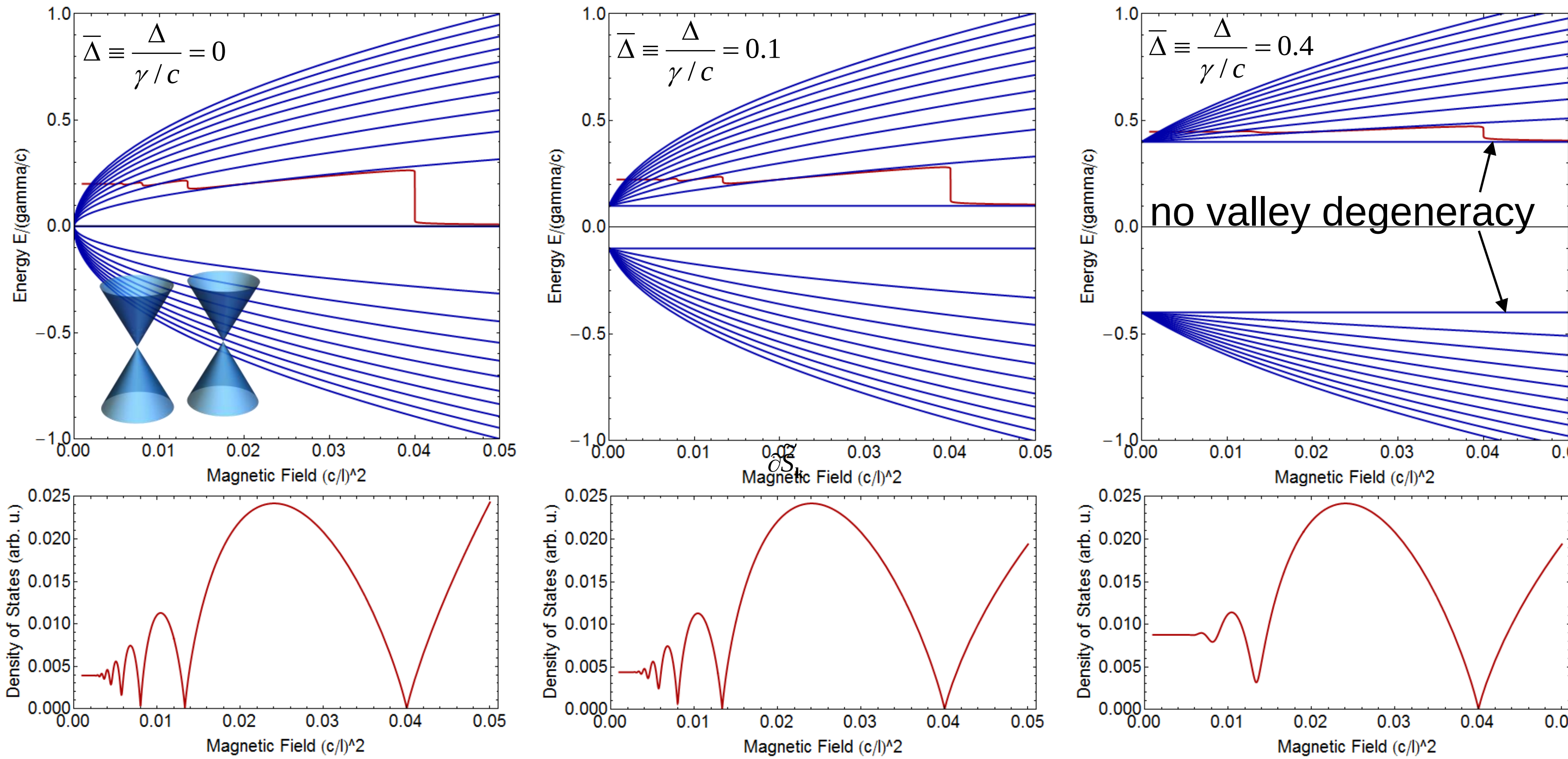
Material	n_0	B_0 (T)	β
Graphite	0.47 ± 0.02	4.52	1/2
GaAs/AlGaAs	0.05 ± 0.01	6.45	0
GaAs (Ref. 20)[a]	0.06 ± 0.02	7.43	0
$Hg_{0.8}Cd_{0.2}Te$ (Ref. 21)[b]	-0.003 ± 0.022	1.298	0
HgTe (Ref. 22)[c]	0.06 ± 0.03	26.14	0
3D AlGaN (Ref. 23)	-0.01 ± 0.03	35.39	0
InSb (Ref. 24)	0.05 ± 0.03	19.80	0
$C_{9.3}AlCl_{3.4}$ (Ref. 25)[d]	0.48 ± 0.02	11.7	1/2





Mass Gap and Shubnikov-de Haas Oscillation

- quantum oscillation in Dirac fermion system with mass gap



SdH phase Φ has no dependence on mass gap Δ.

$$S_{\mathbf{k}}(E) = \pi k^2 = \pi \frac{E^2 - \Delta^2}{\gamma^2} = \frac{2\pi}{l^2}\left(n + \frac{1}{2} - \frac{\Phi}{2\pi}\right)$$

phase correction: $\Phi = \pm\pi$

$$\tilde{S}_{\mathbf{k}}(E) = \pi \tilde{k}^2 = \frac{2\pi}{l^2}\left(n + \frac{1}{2} - \frac{\gamma_n(\tilde{C}_{\mathbf{k}})}{2\pi}\right) \quad (n = 0, 1, 2, \cdots)$$